Appendix 2
Mathematical Interpretation of Historical Process

With regard to social disciplines, a question continually arises: are mathematical methods suitable for analyzing historical and social processes? Obviously, we should not absolutize differences between fields of knowledge, but the division of sciences into two opposite types, made by Wilhelm Windelband and Heinrich Rickert, is still valid. As is known, they singled out sciences involving *nomothetic methods* (*i.e.* searching for general laws and generalizing phenomena) and those applying *idiographic methods* (*i.e.* describing individual and unique events and objects). Rickert attributed history to the second type. In his opinion, history always aims at picturing an isolated and more or less wide course of development in all its uniqueness and individuality (Rickert 1911: 219).

However, since the number of objects and problems investigated and solved by precise methods is growing rapidly, we may assume that, with time, historical knowledge will also be analyzed by some branches of mathematics.

Thus, the problem remains debatable. Nevertheless, rational attempts to use mathematical methods in theoretical or applied trends of the humanities are on the whole positive. Yet, they 'dry up' the soul of history to some extent, but at the same time, they promote self-discipline and self-testing of thoughts, ideas, and concepts of many specialists in the humanities, who, unfortunately, often do not bother to find any methods of testing their conclusions. In addition, this could somewhat reduce the polysemy of the scientific language of the humanities. Rudolf Carnap in his *Philosophical Foundations of Physics* (Carnap 1966) wrote that, even in Physics, the use of terms from ordinary language (as the notion of *law)* for an accurate and nonambiguous expression of ideas complicates proper understanding. However, physicists, as well as other representatives of natural sciences, long ago agreed on fundamentals (such as units of measurement and symbols). As for the humanities, which analyze social phenomena, the same objects sometimes have up to ten meanings and hundreds of definitions. Perhaps, the very necessity to formalize the humanities will lead at last to certain conventions and the ordering of terminology. Nevertheless, even today the use of mathematics may help in searching for a common field of research.

Can we after all construct any mathematical models for such a complex subject of inquiry as the historical process? The answer to this question is obvious: yes, it is quite possible when examining countable objects.

However when we speak about some global general theories, like macroperiodization of the world historical process, any figures, cycles, diagrams and coefficients, of course, cannot prove too much by themselves. Especially, if the respective analysis includes ancient periods for which all the figures are likely to be too much approximate and unreliable. Thus, for general theories covering immense time spans and space, the main proofs are a good empirical basis, logics, internal consistency and productivity of theoretical constructions; that is, a theory's ability to explain the facts better than other theories do. On the other hand, any theory is better when it is supported by more arguments. Mathematical proofs can be rather convincing (when they are relevant, of course). This is especially relevant with respect to those aspects that are more liable to mathematical analysis, for example, those connected with demography.

In this chapter we have chosen such an aspect that is liable to mathematical analysis and quite suitable for it. This is the *temporal* aspect of history. Its suitability for mathematical analysis is connected with the following: though it is quite possible to speak about the tendency of historical time toward acceleration (and this is the subject of the present chapter), the astronomic time remains the same. Thus, within this study we have a sort of common denominator that helps to understand how the ‘numerator’ changes. Hence, we believe that for the analysis of periodization of history the application of mathematical methods is not only possible, but it is also rather productive.

Now we can start our mathematical analysis of the proposed periodization. Mathematical methods are quite widely used in historical research, but, unfortunately, mathematical studies of historical periodization are very few indeed.[1] However, it is worth mentioning that there have been published several issues of the almanac with a speaking title – History and Mathematics (Grinin, de Munck, and Korotayev 2006; Turchin, Grinin, de Munck, and Korotayev 2006; Grinin, Herrmann, Korotayev, and Tausch 2010; Grinin and Korotayev 2014b; Goldstone, Grinin, and Korotayev 2015). In the meantime the discovery of mathematical regularities within an existing periodization may serve as a confirmation of its productivity and as a basis for tentative forecasts. *Time* as a parameter of historical development is quite suitable for mathematical analysis, for example, economic and demographic historians study actively temporal cycles of various lengths (about Juglar and Kondratieff cycles see Grinin and Korotayev 2010a, 2014a; Grinin, Korotayev, and Malkov 2010; Grinin, Korotayev, and Tausch 2016; see also Appendix 3). Cycles used as a basis for this periodization are not different in any principal way from the other temporal cycles with regard to the possibility of being subject to mathematical analysis.

Table 1 ('Chronology of Production Principle Phases') presents dates for all the phases of all the production principles. However, it should be taken into account that in order to make chronology tractable all the dates are approximated even more than the ones used in the text above. Table 2 ('Production Principles and Their Phase Lengths') presents the absolute lengths of the phases in thousands of years.

[1] It appears reasonable to mention here the works by Chuchin-Rusov (2002) and Kapitza (2004b, 2006). Some ideas about the detection of mathematical regularities were expressed by Igor Dyakonov. In particular, he wrote the following: 'There is no doubt that the historical process shows symptoms of exponential acceleration. From the emergence of *Homo Sapiens* to the end of Phase I no less than 30,000 years passed; Phase II lasted about 7,000 years; Phase III – about 2,000, Phase IV – 1,500, Phase V– about 1,000, Phase VI – about 300 years; Phase VII – just over 100 years; the duration of Phase VIII cannot yet be ascertained. If we draw up a graph, these Phases show a curve of negative exponential development' (Dyakonov 1999: 348). However, Dyakonov did not publish the graph. Snooks suggests a diagram called 'The Great Steps of Human Progress' (Snooks 1996: 403; 1998: 208; 2002: 53), which in some sense can be considered as a sort of historical periodization, but this is rather an illustrative scheme for teaching purposes without any explicit mathematical apparatus behind it.

Table 1. Chronology of production principle phases (figures before brackets correspond to absolute datings (BP); figures in brackets correspond to years BCE. **Bold** figures indicate phase lengths (in thousands of years)

Production principle	*1st phase*	*2nd phase*	*3rd phase*	*4th phase*	*5th phase*	*6th phase*	*Overall for production principle*
1. Hunter-Gatherer	40 000–30 000 (38 000–28 000 BCE) **10**	30 000–22 000 (28 000–20 000 BCE) **8**	22 000–17 000 (20 000–15 000 BCE) **5**	17 000–14 000 (15 000–12 000 BCE) **3**	14 000–11 500 (12 000–9500 BCE) **2.5**	11 500–10 000 (9500–8000 BCE) **1.5**	40 000–10 000 (38 000–8000 BCE) **30**
2. Craft-Agrarian	10 000–7300 (8000–5300 BCE) **2.7**	7300–5000 (5300–3000 BCE) **2.3**	5000–3500 (3000–1500 BCE) **1.5**	35000–2200 (1500–200 BCE) **1.3**	2200–1200 (200 BCE–800 CE) **1.0**	800–1430 CE **0.6**	10 000–570 (8000 BCE – 1430 CE) **9.4**
3. Trade-Industrial	1430–1600 **0.17**	1600–1730 **0.13**	1730–1830 **0.1**	1830–1890 **0.06**	1890–1929 **0.04**	1929–1955 **0.025**	1430–1955 **0.525**
4. Scientific-Cybernetic	1955–2000 (1955–1995)* **0.04–0.045**	2000–2040 (1995–2030) **0.035–0.04**	2040–2070 (2030–2055) **0.025–0.03**	2070–2090 (2055–2070) **0.015–0.02**	2090–2105 (2070–2080) **0.01–0.015**	2105–2115 (2080–2090) **0.01**	1955–2115 (2090) [forecast] **0.135–0.160**

Note: In this line figures in brackets indicate the shorter estimates of phases of the Scientific-Cybernetic production principle (the fourth formation). Starting from the second column of this row we give our estimates of the expected lengths of the Scientific-Cybernetic production principle phases.

Table 2. Production principles and their phase lengths (in thousands of years)

Production principle	*1st phase*	*2nd phase*	*3rd phase*	*4th phase*	*5th phase*	*6th phase*	*Overall for production principle*
1. Hunter-Gatherer	10	8	5	3	2.5	1.5	30
2. Craft-Agrarian	2.7	2.3	1.5	1.3	1.0	0.6	9.4
3. Trade-Industrial	0.17	0.13	0.1	0.06	0.04	0.025	0.525
4. Scientific-Cybernetic	0.04–0.045	0.035–0.04*	0.025–0.03	0.015–0.02	0.01–0.015	0.01	0.135–0.160

Note: * This line indicates our estimates of the expected lengths of the Scientific-Cybernetic production principle phases.

Table 3 ('Ratio of Each Phase [and Phase Combination] Length to the Total Length of Respective Production Principle [%%]') presents results of our calculations of the ratio of each phase's length to the length of the respective production principle using a rather simple methodology.[2] Table 4 ('Comparison of Phase Length Ratios for Each Production Principle [%%]') employs an analogous methodology to compare lengths of phases (and combinations of phases) within one production principle. For example, for the hunter-gatherer production principle the ratio of the first phase length (10,000 years) to the second one (8,000 years) equals 125 per cent; whereas the ratio of the second phase to the third one (5,000 years) is 160 per cent. In the meantime the ratio of the sum of the first and the second phases' lengths to the sum of the third and the fourth phases (3,000 years) equals 225 per cent. Tables 3 and 4 also present the average rates for all the production principles.

[2] The absolute length of a phase (or a sum of the lengths of two or three phases) is divided by the full length of the respective production principle. For example, if the length of the hunter-gatherer production principle is 30,000 years, the length of its first phase is 10,000, the one of the second is 8,000, the duration of the third is 5,000, then the ratio of the first phase length to the total production principle length will be 33.3 per cent; the ratio of the sum of the first and the second phases' lengths to the total production principle length will be 60 per cent; and the ratio of the sum of the first, the second, and the third phases' lengths to the total production principle length will be 76.7 per cent.

Table 3. Ratio of each phase (and phase combination) length to the total length of respective production principle (%%)

Production principle	*1*	*2*	*3*	*4*	*5*	*6*	*1–2*	*3–4*	*5–6*	*1–3*	*4–6*
1. Hunter-Gatherer	33.3	26.7	16.7	10	8.3	5	60	26.7	13.3	76.7	23.3
2. Craft-Agrarian	28.7	24.5	16.0	13.8	10.6	6.4	53.2	29.8	17	69.1	30.9
3. Trade-Industrial	32.4	24.8	19	11.4	7.6	4.8	57.1	30.5	12.4	76.2	23.8
4.Scientific-cybernetic	28.1 (29.6)*	25 (25.9)	18.8 (18.5)	12.5 (11.1)	9.4 (7.4)	6.3 (7.4)	53.1 (55.6)	31.3 (29.6)	15.6 (14.8)	71.9 (74.1)	28.1 (25.9)
Average	30.6**	25.3	17.6	11.9	9	5.6	55.9	29.6	14.6	73.5	26.5

Note: * In this line figures in brackets indicate the shorter estimates of the scientific-cybernetic production principle's phases (the fourth formation).
** The calculation of average value took into account only one version of the scientific-cybernetic production principle evolution (that is the figures before the brackets).

Table 4. Comparison of phase length ratios for each production principle (%%)

Production principle	*1:2*	*2:3*	*3:4*	*4:5*	*5:6*	*(1+2):(3+4)*	*(3+4):(5+6)*	*(1+2+3):(4+5+6)*
1. Hunter-Gatherer	125	160	166.7	120	166.7	225	200	328.6
2. Craft-Agrarian	117.4	153.3	115.4	130	166.7	178.6	175	224.1
3. Trade-Industrial	130.8	130	166.7	150	160	187.5	246.2	320
4. Scientific-Cybernetic	112.5 (114.3)	133.3 (140)	150 (166.7)	133.3 (150)	150 (100)	170 (187.5	200 (200)	255.5 (285.7)
Average*	121.4	144.2	149.7	133.3	160.9	190.3	205.3	282.1

Note: * The calculation of average value took into account only one version of the scientific-cybernetic production principle evolution (that is the figures before the brackets).

Thus, the proposed periodization is based on the idea of recurrent developmental cycles (each of them includes six phases); however, each subsequent cycle is shorter than the previous one due to the acceleration of historical development. No doubt that these are recurrent cycles, because within each cycle in some respect development follows the same pattern: every phase within every cycle plays a functionally similar role; what is more, the proportions of the lengths of the phases and their combinations remain approximately the same (see Tables 3 and 4). All this is convincingly supported by the above mentioned calculations, according to which stable proportions of the lengths of phases and their combinations remain intact with the change of production principles.

In general, our mathematical analysis represented in diagrams and tables indicates the following points: a) evolution of each production principle in time has recurrent features, as is seen in Diagrams 1–4; b) there are stable mathematical proportions between the lengths of phases and phase combinations within each production principle (Tables 3 and 4); c) the cycle analysis clearly indicates that the development speed increases sharply just as a result of production revolutions (see Diagram 5); d) if we calibrate the Y-axis of the diagram,[3] the curve of historical process acquires a hyperbolic (Diagram 6) rather than exponential shape (as in Diagrams 1–4), which indicates that we are dealing here with a blow-up regime (Kapitza, Kurdjumov, and Malinetskij 1997).

[3] Within the calibrated scale the changes from one production principle to another are considered as changes by an order of magnitude, whereas changes within a production principle are regarded as changes by units within the respective order of magnitude. Such a calibration appears highly justified, as it does not appear reasonable to lay off the same value at the same scale both for the transition from one principle of production to another (for example, for the Agrarian Revolution), and for a change within one production principle (*e.g.*, for the development of specialized intensive gathering). Indeed, for example, the former shift increased the carrying capacity of the Earth by an order or two magnitude, whereas the latter led to the increase of carrying capacity by two-three times at best.

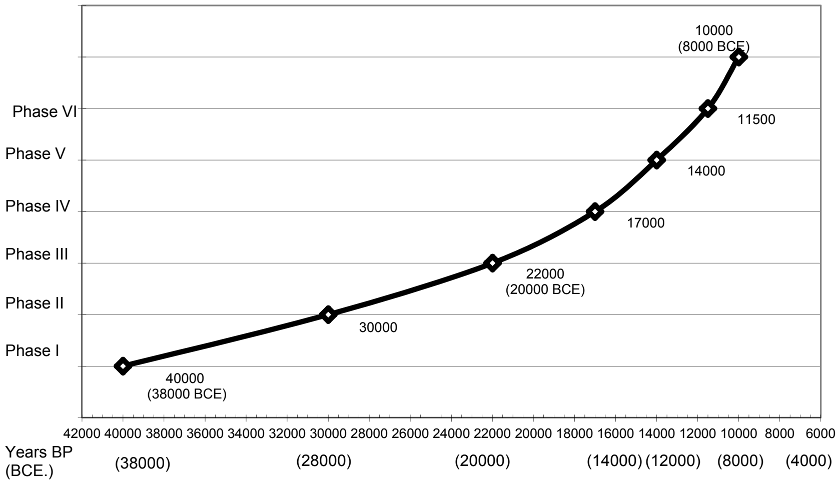

Diagram 1. Hunter-Gatherer production principle

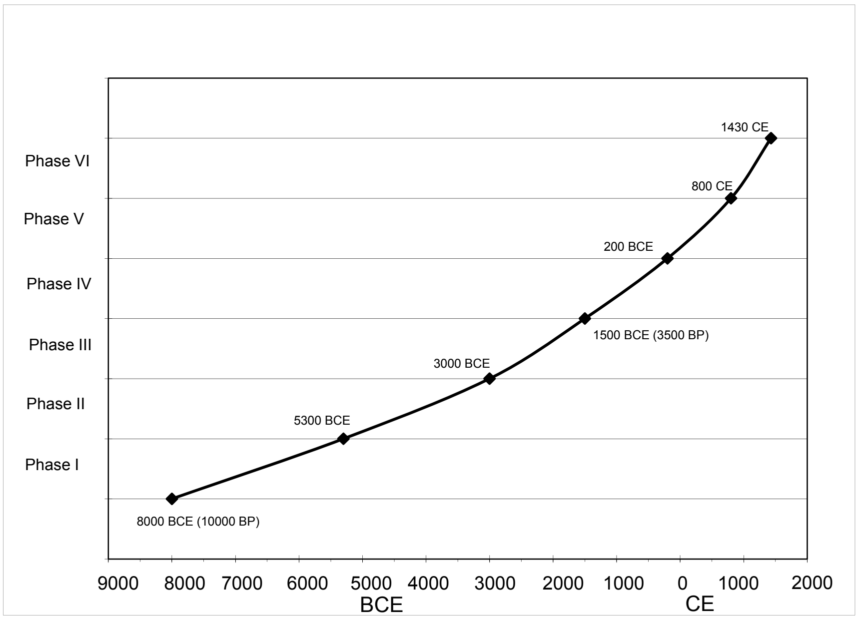

Diagram 2. Craft-Agrarian production principle

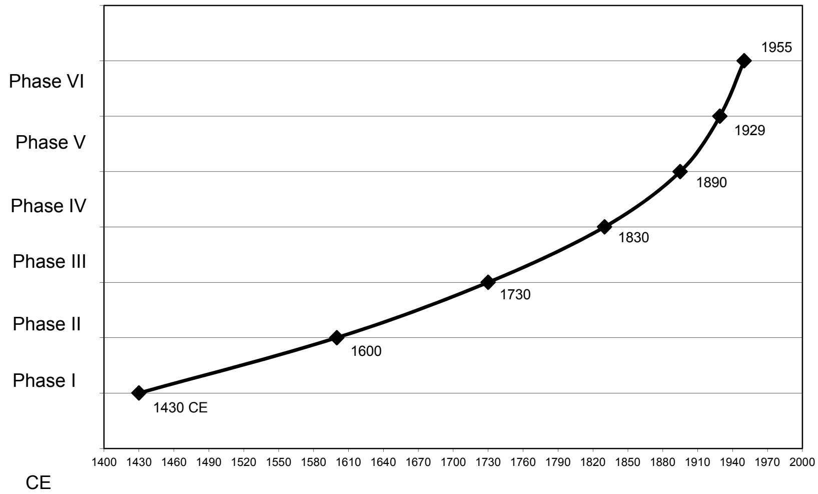

Diagram 3. Trade-Industrial production principle

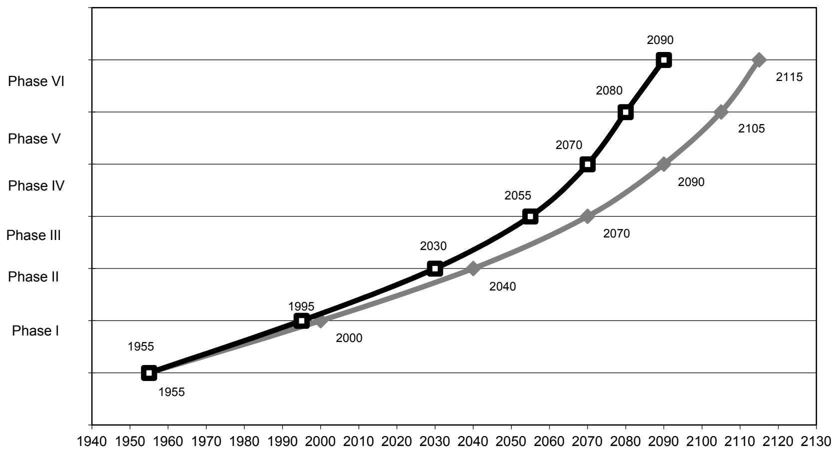

Diagram 4. Scientific-Cybernetic production principle

Note: The broken line indicates the forecast version for the expected development of the scientific-cybernetic production principle corresponding to the dates in the brackets in the line of the scientific-cybernetic production principle in Table 1.

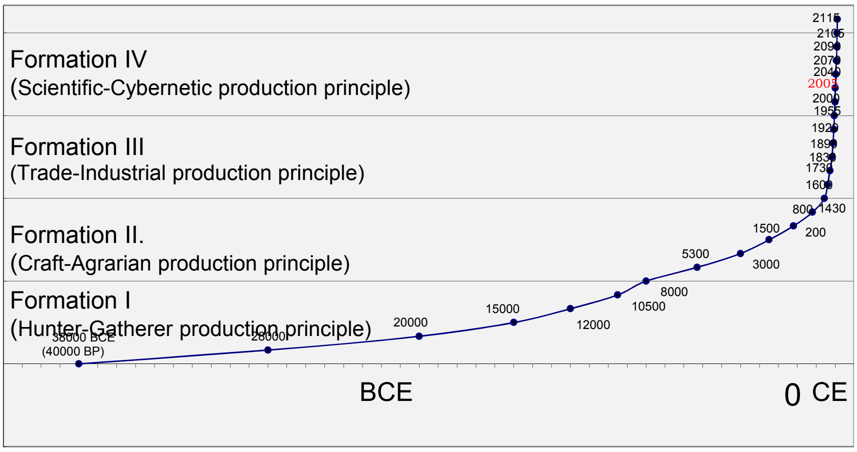

Diagram 5. Evolution of historical process in time

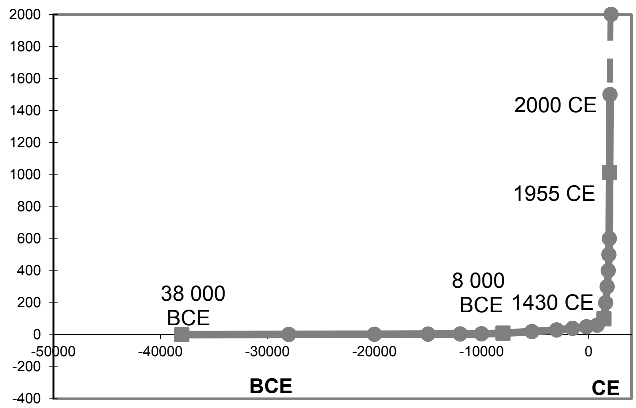

Diagram 6. Hyperbolic model of historical process dynamics

The analysis of stable proportions of production principle cycles makes it possible to propose some tentative forecasts (in particular, with respect to the lengths of the remaining phases of the fourth production principle).

And the last comment. The historical process curve (see Diagram 6) might look a bit embarrassing, as it goes to infinity within a finite period of time. In this respect Dyakonov (1999: 348) notes the following:

> As applied to history, the notion [of infinity] seems to make no sense: the succession of Phases, their development ever more rapid, cannot end in changes taking place every year, month, week, day, hour or second. To avoid a catastrophic outcome – let us hope that wise *Homo Sapiens* will find a way – then we have to anticipate intervention from as yet unknown forces.

However, it should be taken into account that the diagram depicts the development of just one variable of the historical process – the technological one, whereas the high correlation between general development and technological development is observed within certain limits. Outside these limits various deviations (both with respect to the development vectors and its speed) are possible. First of all, it is quite evident that the general development of the system does not catch up with the technological one; secondly, the growing gap implies that the price for progress will grow too. In other words, uncontrolled scientific-technological and economic changes lead to the growth of various deformations, crisis phenomena in various spheres of life, which slow down the overall movement and in many respects change its direction. Actually, if the system persists, the overall speed of its development cannot exceed the speed of the least dynamic (most conservative) element (*e.g.*, religious-ideological consciousness, law) whose change needs the change of generations. The growth of the system gaps in connection with changing economic, information, and technological realities can lead to its breakdown and its replacement with another system. And the price paid for such a rapid transformation of such an immensely complex system as modern humankind may be very high indeed.

There would be even no future shock that Toffler discovered for public in 1970 (Toffler 1970). This implies an immense acceleration of development that can be hardly compatible with the biopsychic human nature. Indeed, in view of the growing life expectancies all the immense changes (the 2040s to 2090s) will happen within the span of one generation that will appear in the 2010s. The significance of these changes will be no smaller (what is more, it is likely to be greater) than the significance of the ones that took place between 1830 and 1950 that included gigantic technological transformations, the transition from agricultural to industrial society, social catastrophes and world wars. However, these metamorphoses took place within 120 years, whereas the expected period of the forthcoming transformation is twice as short. And if they occur within a lifespan of one generation, it is not clear whether human physical and psychic abilities will be sufficient to stand this; what will be the cost of such a fast adaptation? Thus, we confront the following question: how could the gap between the development of productive forces and other spheres of life is compensated?[4] Besides one should take into account the point that precisely this generation will have the 'controlling stake' of votes during elections (taking into account the fertility decline that is likely to continue throughout this century), and it is not clear if this generation will be able to react adequately to the rapidly changing environment.

In this context the issue of preserving the purely human nature in the man of the future (which we spoke about in *Introduction*, Chapter 3, and *Afterword*) becomes particularly urgent.

But here one should add that the global ageing increases the conservatism of the elderly generation (see also *Afterword*); thus, ageing can become a brake which will adjust the rate of changes to human psychophysiological capabilities. Yet, there are also some trends to slow down this development. Sergey Tsirel (2008) pays attention to one of them. He points out that 'ordinary' time (*i.e.* everyday and common tempos and rhythm within the limits of usual human existence) starts to hamper historical changes, because it is hard for people to break themselves of the habit of what they got accustomed in childhood and youth, they deliberately or unconsciously resist changes in various ways. Actually one may completely agree that such a sound conservatism is able to prevent acceleration. However, will not this conservatism become the cause of great problems in other spheres (see *Afterword*)?

Thus, it is obvious that our civilization stands at the threshold of dramatic seminal transformations which require being more careful on the way to further technological and social changes.

[4] On the acceleration of historical time and the necessity of stabilization see Grinin 1998a, 2006a; see also Dyakonov 1999: 348; Kapitza 2004a, 2004b, 2006; Korotayev, Malkov, and Khaltourina 2006a, 2006b.