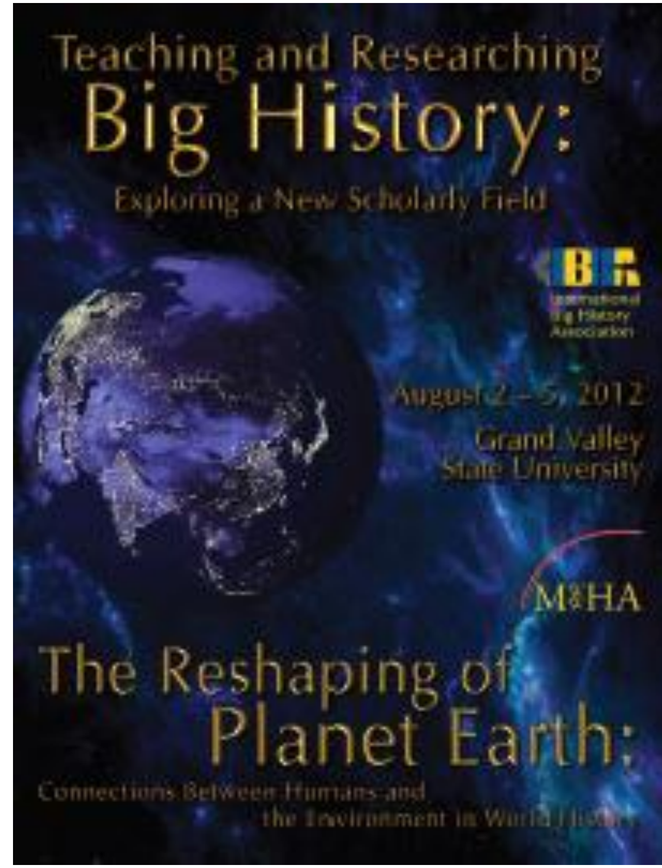
MATHEMATICAL MODELING OF BIOLOGICAL & SOCIAL PHASES OF BIG HISTORY



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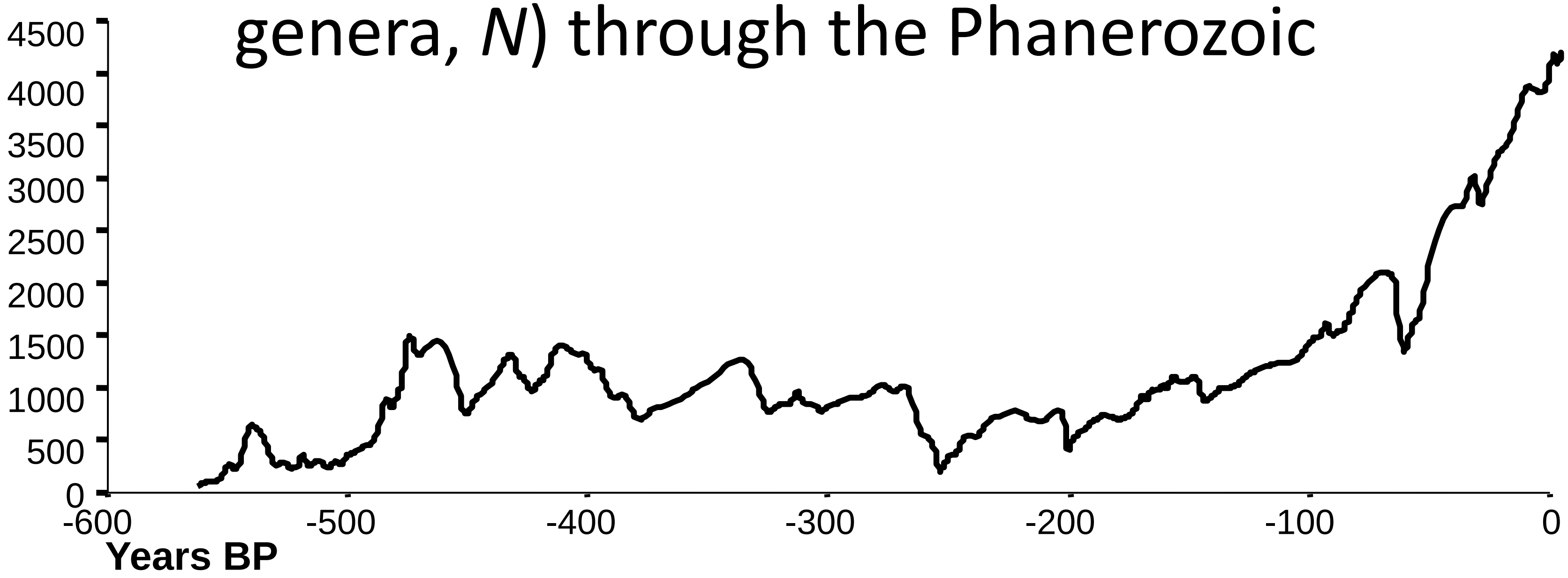
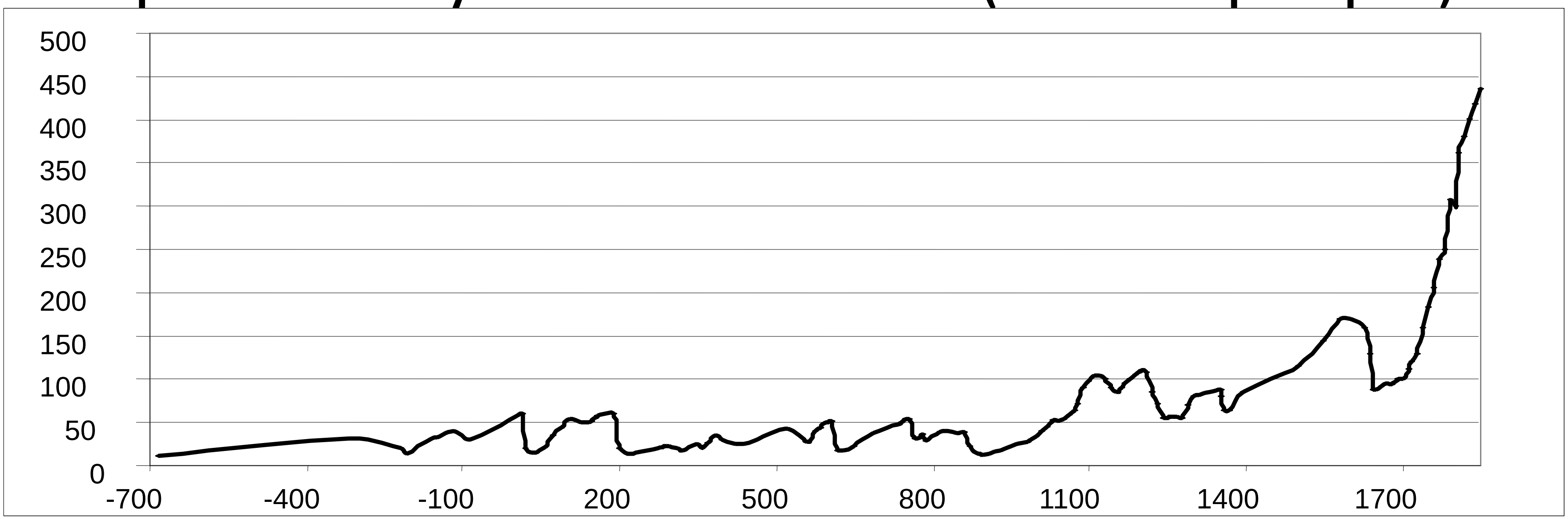
Global change in marine biodiversity (number of genera, *N*) through the Phanerozoic



Population dynamics of China (million people)



- **In 1960 Heinz von Foerster and his colleagues, Mora, and Amiot published, in the journal *Science*, a striking discovery.**

Von Foerster showed that between 1 and 1958 CE the world's population (*N*) dynamics can be described in an extremely accurate way with an astonishingly simple equation:

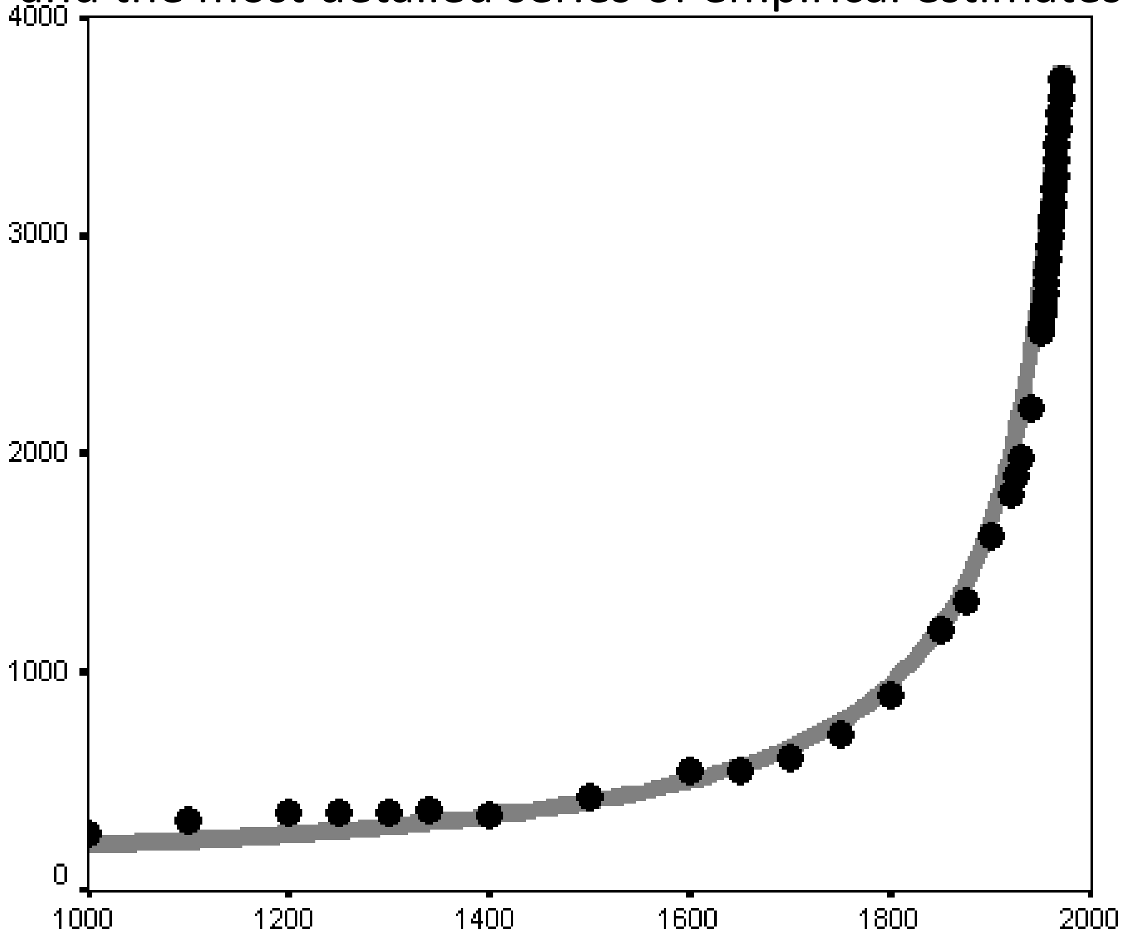
$$N_t = \frac{C}{t_0 - t}$$

where N_t is the world population at time t, and C and t_0 are constants, with t_0 corresponding to an absolute limit ("singularity" point) at which N would become infinite.

Parameter t_0 was estimated by von Foerster and his colleagues as 2026.87, which corresponds to November 13, 2026; this made it possible for them to supply their article with a public-relations masterpiece title:

"Doomsday: Friday, 13 November, A.D. 2026"

The overall correlation between the curve generated by von Foerster's equation and the most detailed series of empirical estimates looks as follows:



The formal characteristics are as follows:
$R = 0.998$; $R^2 = 0.996$.

Thus, von Foerster's equation accounts for an astonishing 99.6% of all the macrovariation in world population, from 1000 CE through 1970.

Note also that the empirical estimates of world population find themselves aligned in an extremely neat way along the hyperbolic curve, which convincingly justifies the designation of the pre-1970s world population growth pattern as "hyperbolic".

To start with, the von Foerster equation,

$$N_t = \frac{C}{t_0 - t}$$

is just the solution for the following differential equation:

$$\frac{dN}{dt} = \frac{N^2}{C}$$

This equation can be also written as:

$$\frac{dN}{dt} = aN^2,$$ where $$a = \frac{1}{C}$$

$$\frac{dN}{dt} = aN^2$$

In our context *dN/dt* denotes the absolute population growth rate at some moment of time. Hence, this equation states that the absolute population growth rate at any moment of time should be proportional to the square of population at this moment.

Note that this significantly demystifies the problem of the world population hyperbolic growth. Now to explain this hyperbolic growth we should just explain why for many millennia the world population's absolute growth rate tended to be proportional to the square of the population.

The existing mathematical models of the World System hyperbolic growth are based on the following assumptions:

1) First of all the Malthusian assumption is made “that population is limited by the available technology, so that the growth rate of population is proportional to the growth rate of technology" (Kremer 1993: 681–2).

This statement looks quite convincing.

Indeed, throughout most of human history the world population was limited by the technologically determined ceiling of the carrying capacity of land.

E.g., with foraging subsistence technologies the Earth could not support more than 10 million people, because the amount of naturally available useful biomass on this planet is limited, and the world population could only grow over this limit when the people started to apply various means to increase artificially the amount of available biomass, that is with the transition from foraging to food production.

However, the extensive agriculture also can only support a limited number of people, and further growth of the world population became only possible with the intensification of agriculture and other technological improvements.

On the other hand, as is well known, the technological level is not a constant, but a variable. And in order to describe its dynamics the second basic assumption is employed:

"High population spurs technological change because it increases the number of potential inventors…

In a larger population there will be proportionally more people lucky or smart enough to come up with new ideas" (Kremer 1993: 685), thus, "the growth rate of technology is proportional to total population".

In general, we find this assumption rather plausible – in fact, it is quite probable that, other things being equal, within a given period of time, one billion people will make approximately one thousand times more inventions than one million people.

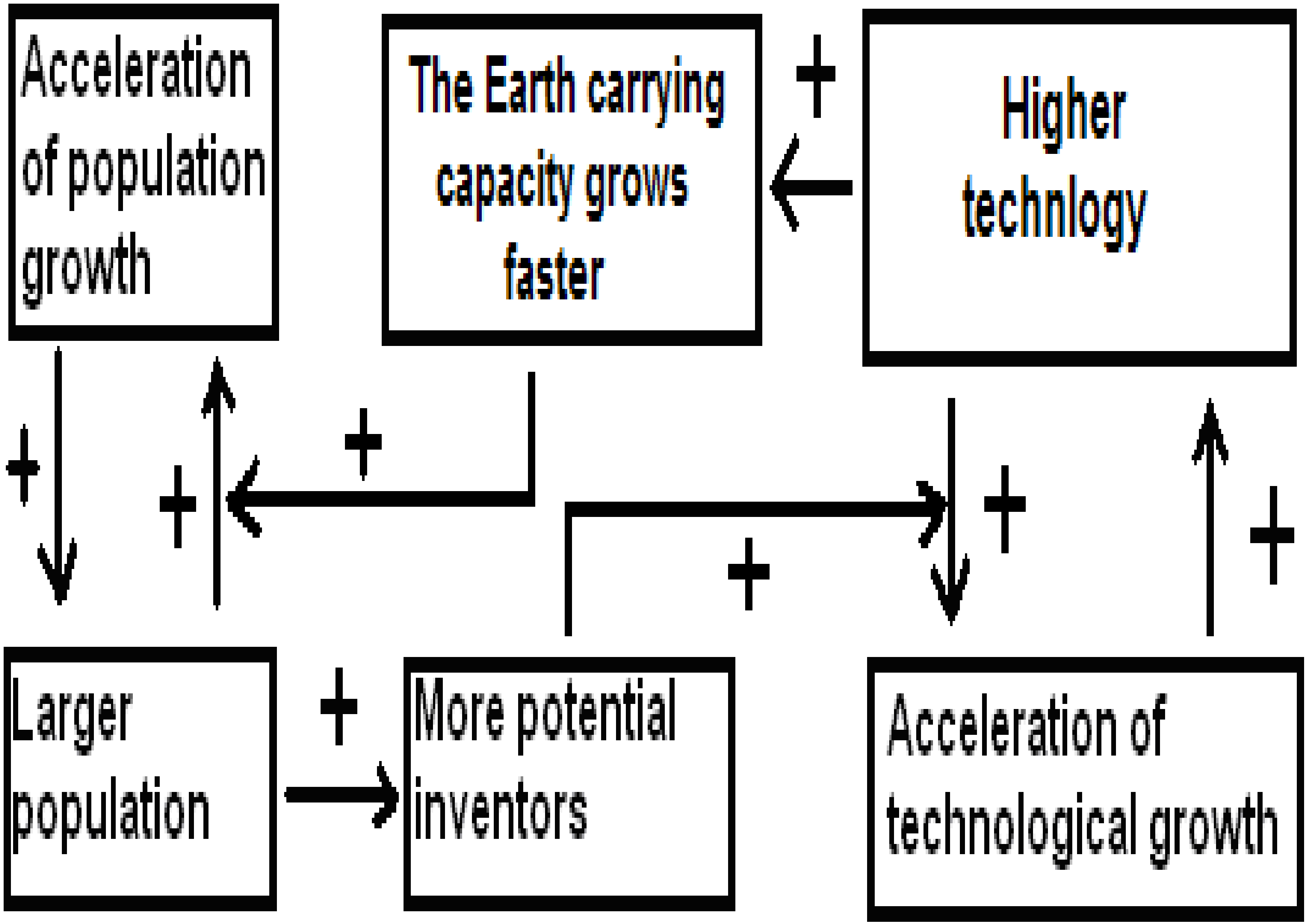
This assumption is expressed mathematically in the following way:

$$\frac{dT}{dt} = bNT$$

Actually, this equation just says that the absolute technological growth rate $\frac{dT}{dt}$ at a given moment of time is proportional to the technological level (T) observed at this moment (the wider is the technological base, the more inventions could be made on its basis), and, on the other hand, it is proportional to the population (the larger the population, the higher the number of potential inventors).

In fact, the described model suggests that the hyperbolic pattern of the world's population growth could be accounted for by the nonlinear second order positive feedback mechanism that was shown long ago to generate just the hyperbolic growth, known also as the "blow-up regime".

In our case this nonlinear second order positive feedback looks as follows: the more people – the more potential inventors – the faster technological growth – the faster growth of the Earth's carrying capacity – the faster population growth – with more people you also have more potential inventors – hence, faster technological growth, and so on.

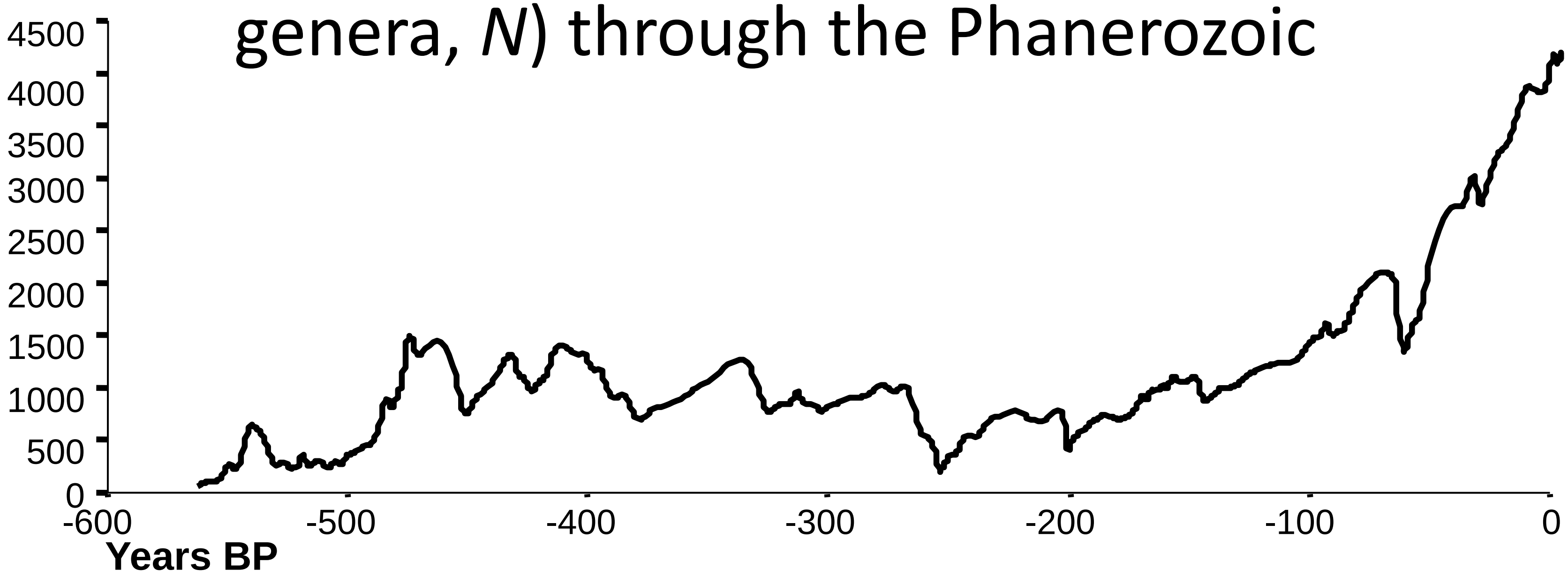

Acceleration of population growth
The Earth carrying capacity grows faster
Higher technlogy
Larger population
More potential inventors
Acceleration of technological growth
+
+
+
+
+
+
+
+
+

The described model provides a rather convincing explanation of *why* throughout most of human history the world population followed the hyperbolic pattern with the absolute population growth rate tending to be proportional to N^2.

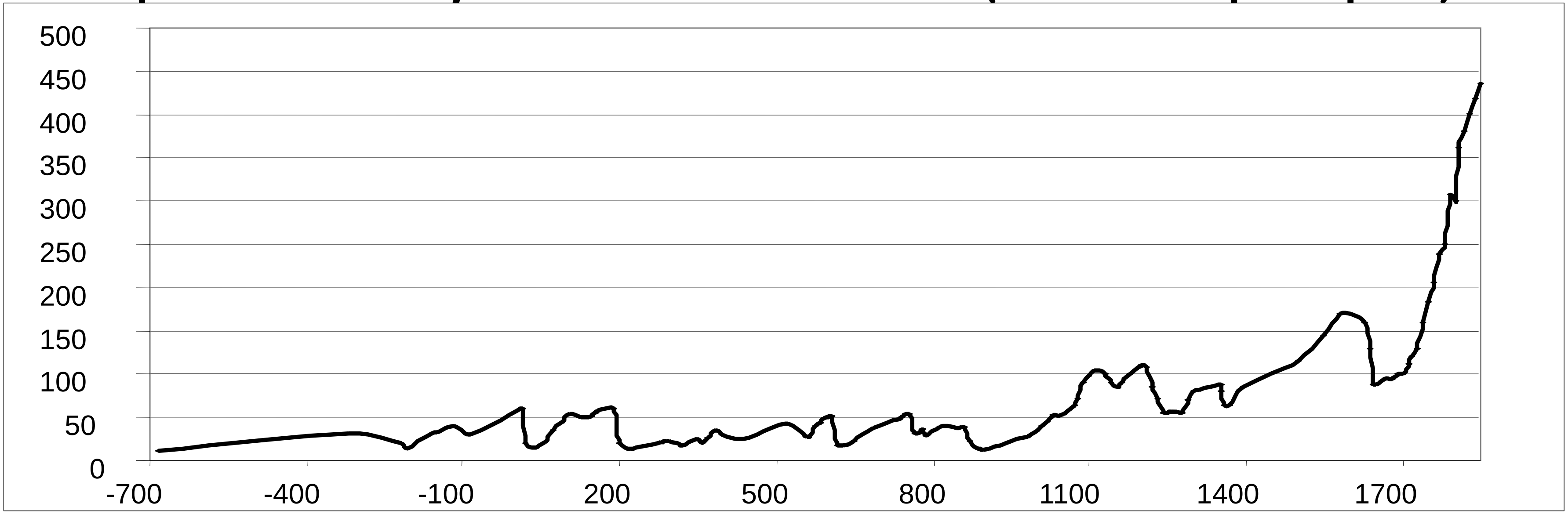
For example, why will the growth of population from, say, 10 million to 100 million, result in the growth of *dN/dt* 100 times? The described model explains this rather convincingly. The point is that the growth of world population from 10 to 100 million implies that human technology also grew approximately 10 times (given that it will have proven, after all, to be able to support a ten times larger population).

Note also that **the process discussed above should be identified with the process of collective learning**. Respectively, mathematical models of the World System development discussed in this article can be interpreted as mathematical models of the influence of collective learning on the global social evolution. Thus, a rather peculiar hyperbolic shape of the acceleration of the global development observed prior to the early 1970s may be regarded just as a product of the global collective learning.

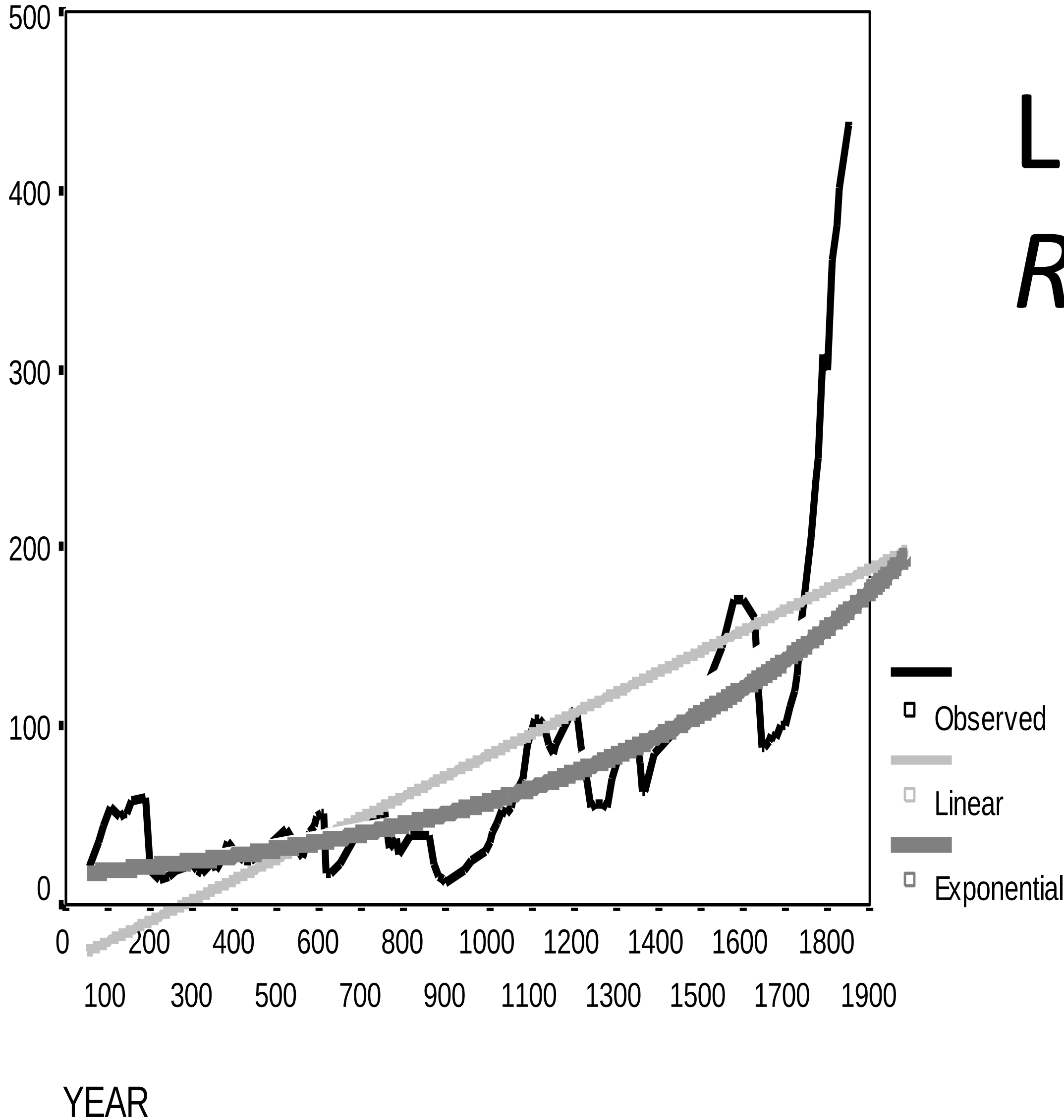
Global change in marine biodiversity (number of genera, *N*) through the Phanerozoic



Population dynamics of China (million people)



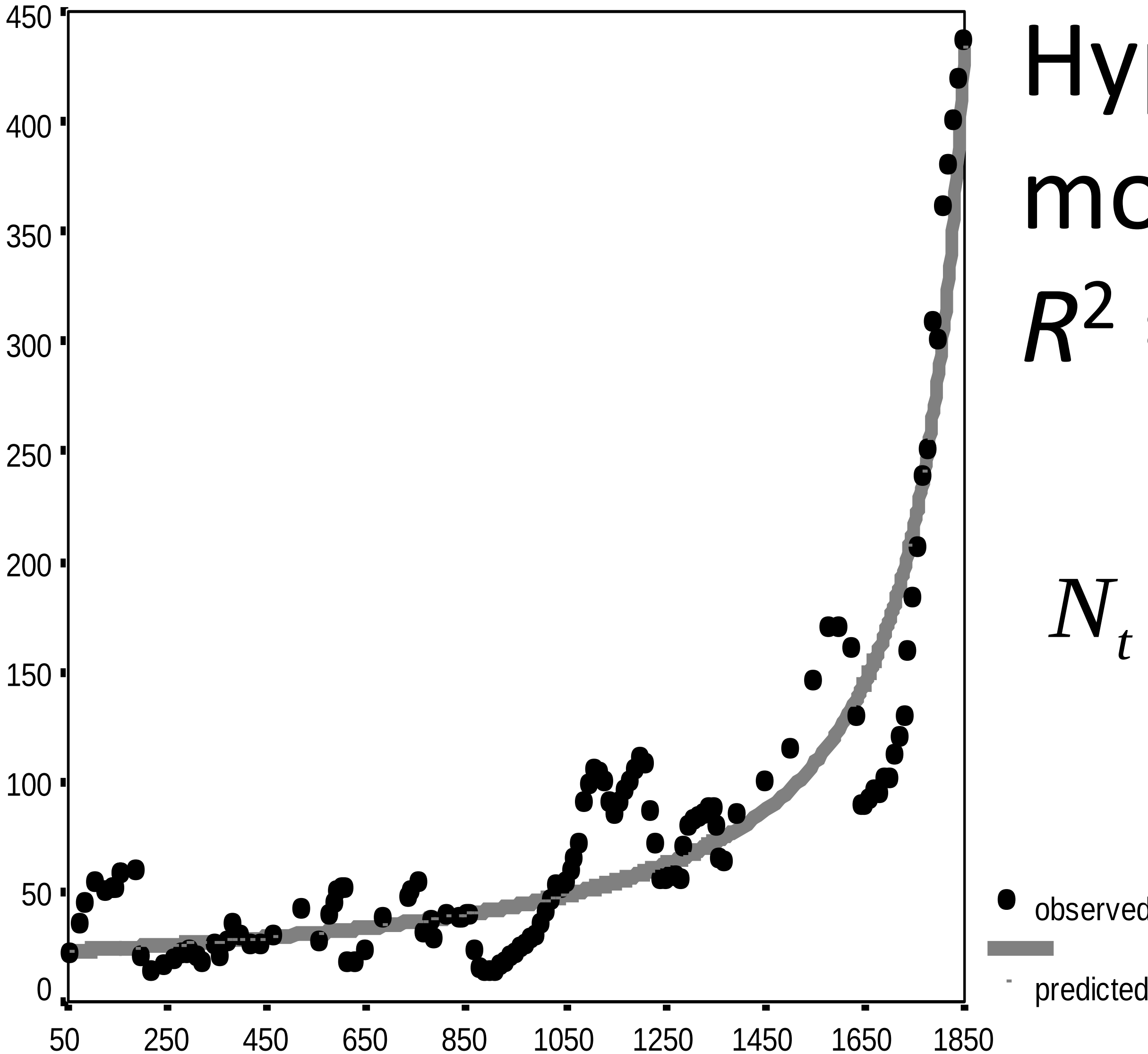
Population dynamics of China (million people), 57-1851 CE



Linear model:
$R^2 = 0.469$

Exponential model:
$R^2 = 0.600$

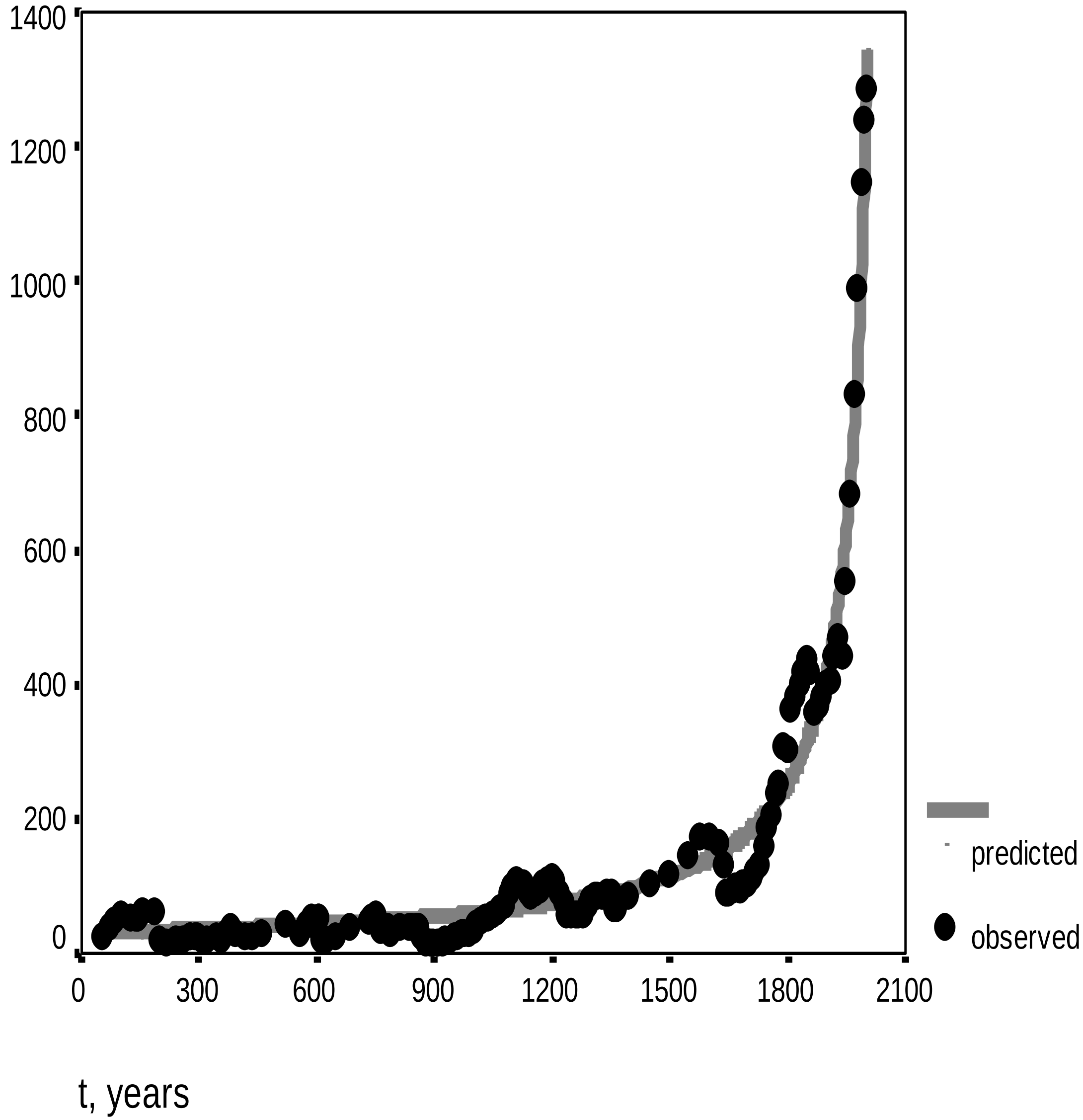
Population dynamics of China (million people), 57-1851 CE



Hyperbolic model:
$R^2 = 0.884$

$$N_t = \frac{33430{,}518}{1915 - t}$$

Population dynamics of China (million people), 57-2003 CE

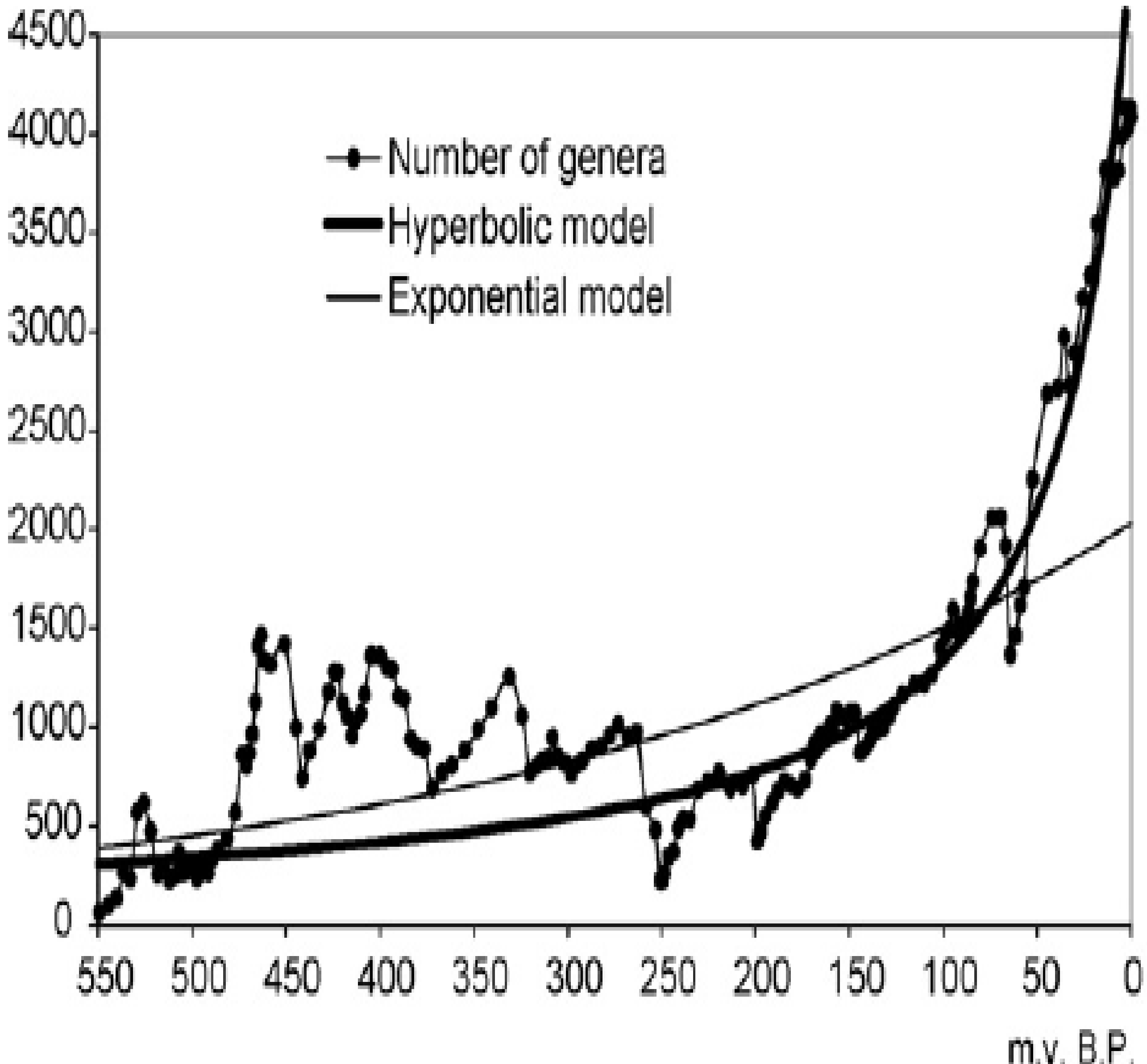


Hyperbolic model:

$R^2 = 0.968$

$$N_t = \frac{63150{,}376}{2050 - t}$$

Global change in MARINE biodiversity (number of GENERA, *N*) through the Phanerozoic

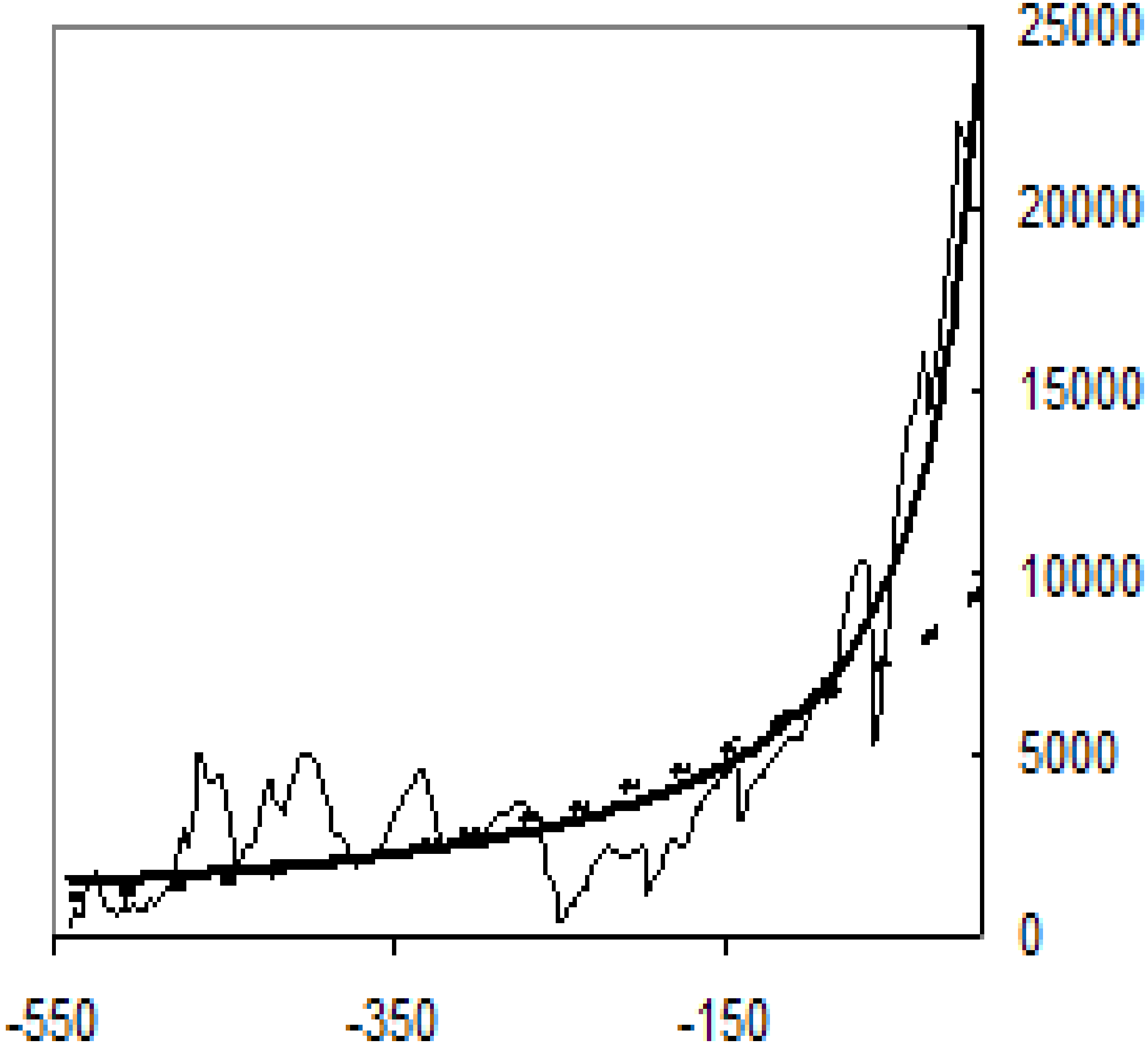


Exponential model:
$R^2 = 0.463$

Hyperbolic model:
$R^2 = 0.854$

$$N_t = \frac{183320}{37 - t}$$

Global change in MARINE biodiversity (number of SPECIES, *N*) through the Phanerozoic

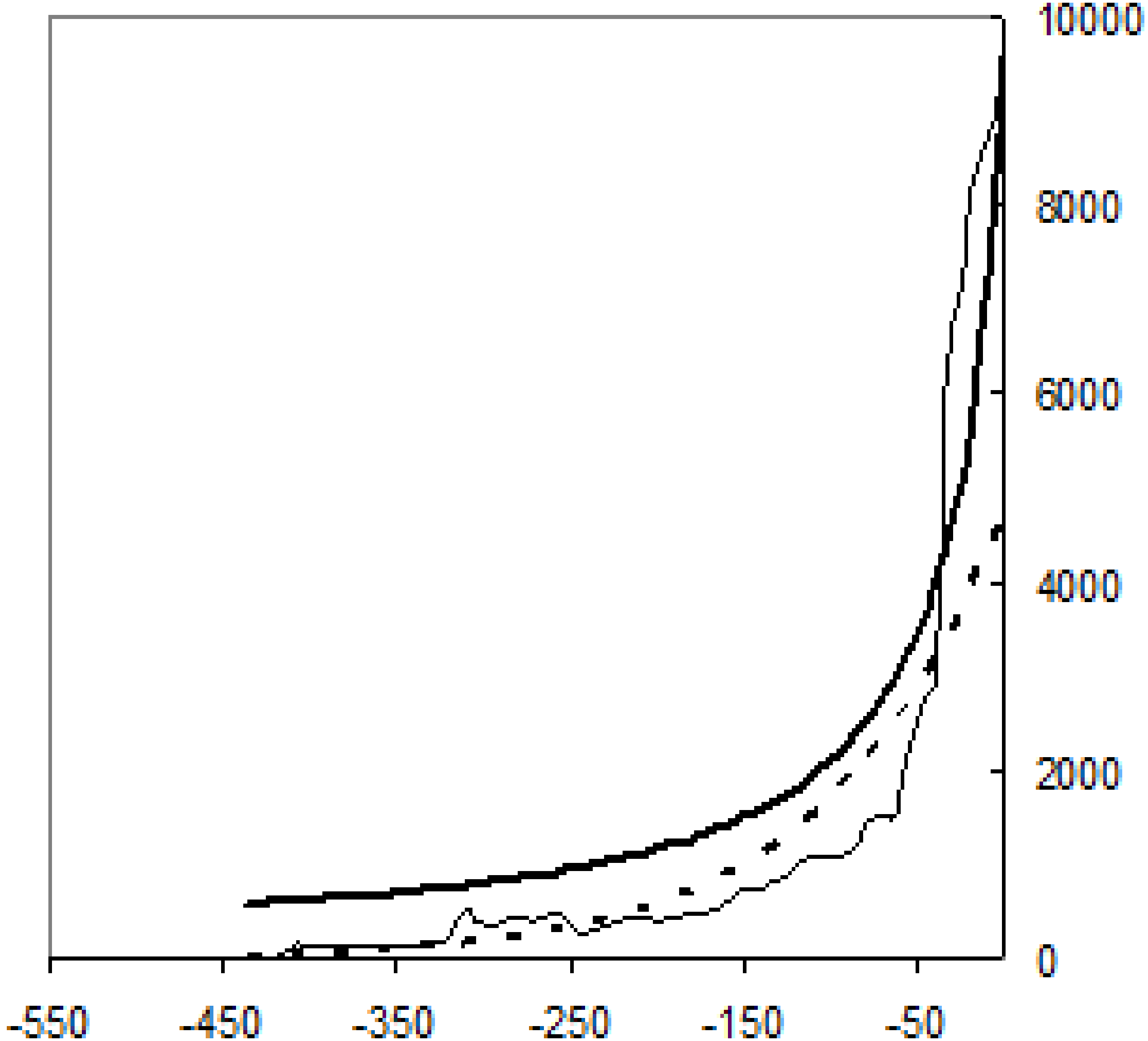


Exponential model:
$R^2 = 0.51$

Hyperbolic model:
$R^2 = 0.91$

$$N_t = \frac{892874}{35 - t}$$

Global change in CONTINENTAL biodiversity (number of GENERA, *N*) through the Phanerozoic

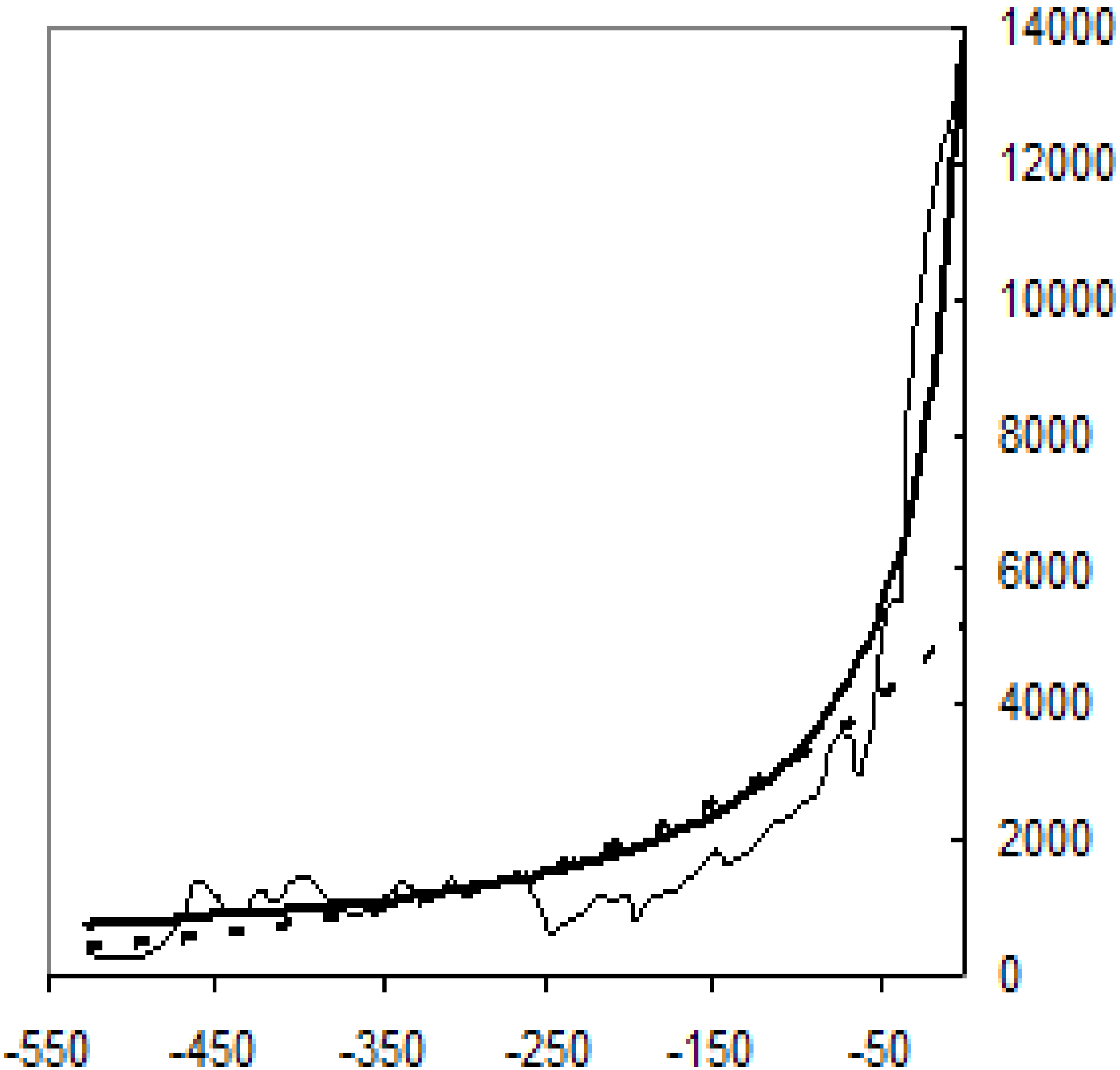


Exponential model:
$R^2 = 0.86$

Hyperbolic model:
$R^2 = 0.94$

$$N_t = \frac{272095}{29-t}$$

Global change in TOTAL (MARINE + CONTINENTAL) biodiversity (number of GENERA, *N*)



Exponential model:
$R^2 = 0.67$

Hyperbolic model:
$R^2 = 0.95$

$$N_t = \frac{434635}{30 - t}$$

In the macrosociological models, the hyperbolic pattern of the world population growth arises from a non-linear second-order positive feedback between the demographic growth and technological development (more people - more potential inventors - faster technological growth - the carrying capacity of the Earth grows faster - faster population growth - more people - more potential inventors, and so on).

Based on the analogy with macrosociological models and diverse paleontological data, we suggest that the hyperbolic character of biodiversity growth can be similarly accounted for by a non-linear second-order positive feedback between the diversity growth and community structure complexity.

The feedback can work via two parallel mechanisms:

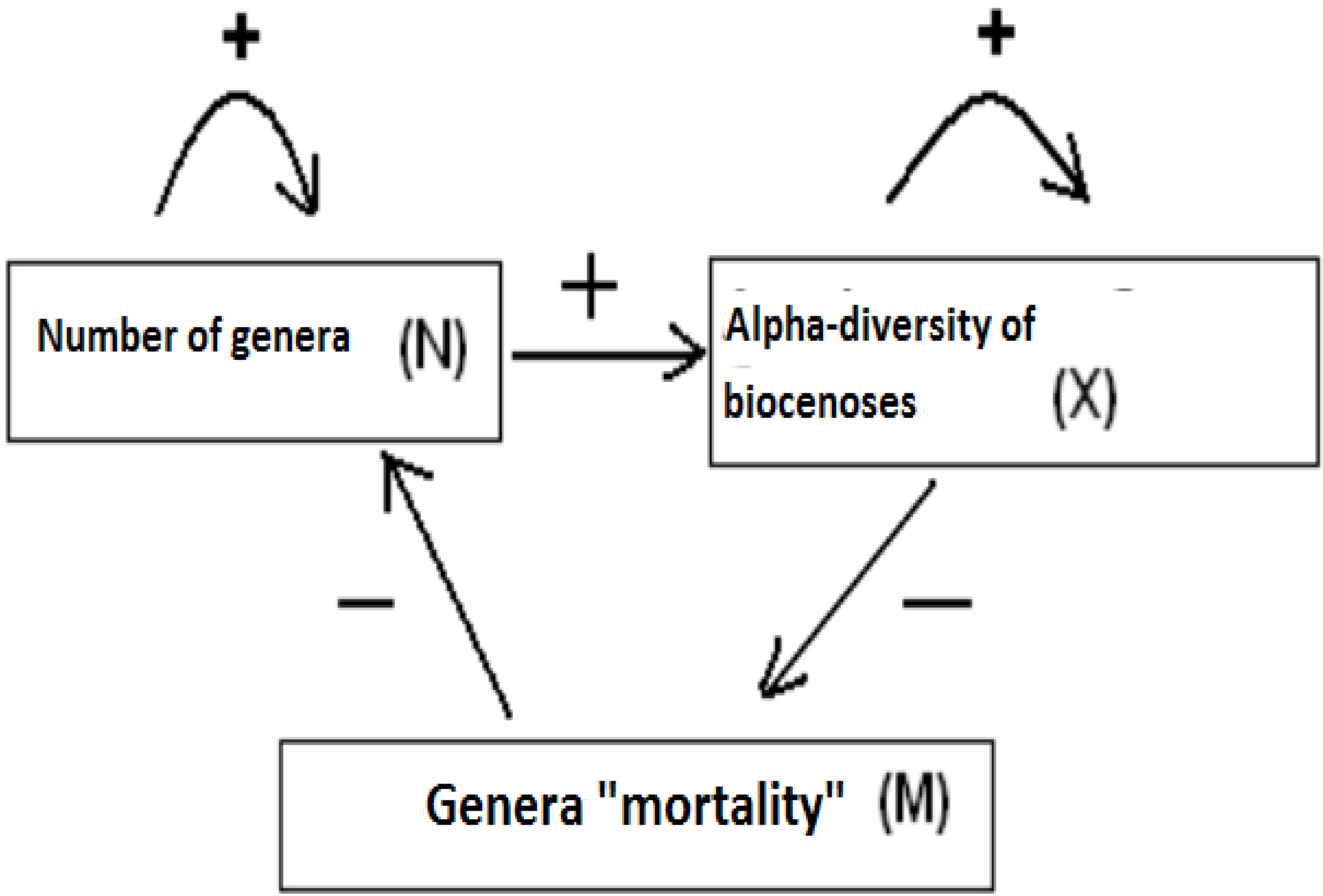
1) decreasing extinction rate (more taxa- higher alpha diversity, or mean number of taxa in a community - communities become more complex and stable - extinction rate decreases - more taxa, and so on) and
2) increasing origination rate (new taxa facilitate niche construction; newly formed niches can be occupied by the next "generation" of taxa).

The latter possibility makes the mechanisms underlying the hyperbolic growth of biodiversity and human population even more similar, because the total ecospace of the biota is analogous to the "carrying capacity of the Earth" in demography.

The hyperbolic growth of the Phanerozoic biodiversity suggests that "cooperative" interactions between taxa can play an important role in evolution, along with generally accepted competitive interactions. Due to this "cooperation", the evolution of biodiversity acquires some features of a self-accelerating process.

Macroevolutionary "cooperation" reveals itself in:

1) increasing stability of communities that arises from alpha diversity growth;
2) ability of species to facilitate opportunities for additional species entering the community.


+
+
+
Number of genera (N)
Alpha-diversity of biocenoses (X)
−
−
Genera "mortality" (M)

One wonders if it cannot be regarded as a (rather imperfect) analogue of the collective learning mechanism that plays such an important role within the social macroevolution.

$$\frac{dN}{dt} = a(bK - N)N$$

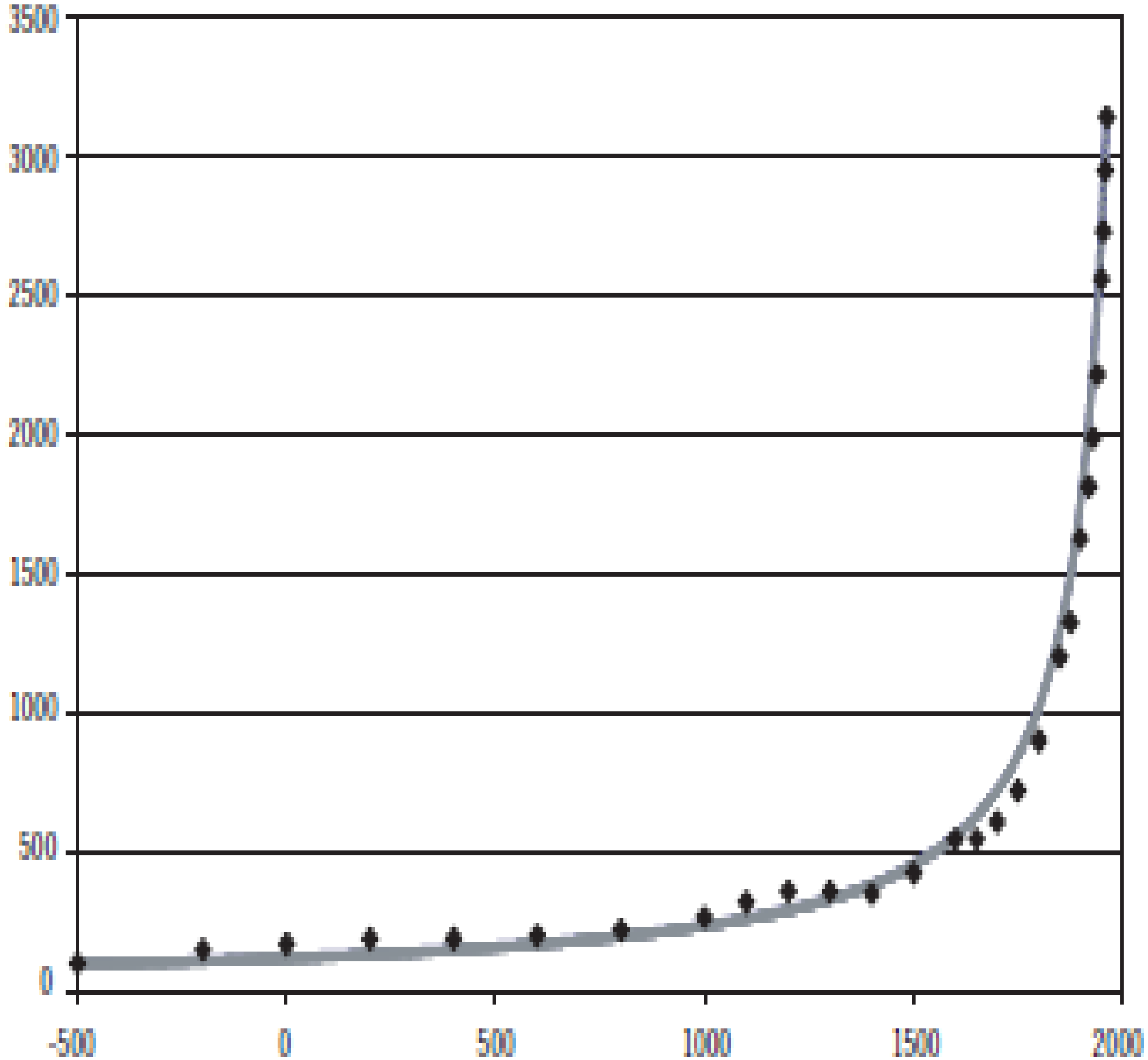
$$\frac{dK}{dt} = cNK$$

where *N* is the world population,

K is the level of technology;

bK corresponds to the number of people (*N*), which the earth can support with the given level of technology (*K*).

Predicted and Observed Dynamics of World Population Growth, in millions (500 BCE – 1962 CE)



$R^2 = 0.9966$

$$\frac{dN}{dt} = k(N_{\max} - N)N$$

where Nmax corresponds to ecospace.

$$\frac{dN}{dt} = k_1(N_{\max} - N)N$$

$$\frac{dN_{\max}}{dt} = k_2 XN$$

$$\frac{dX}{dt} = k_3 NX$$

where X corresponds to alpha-diversity.

$$\frac{dN}{dt} = k_4 XN$$

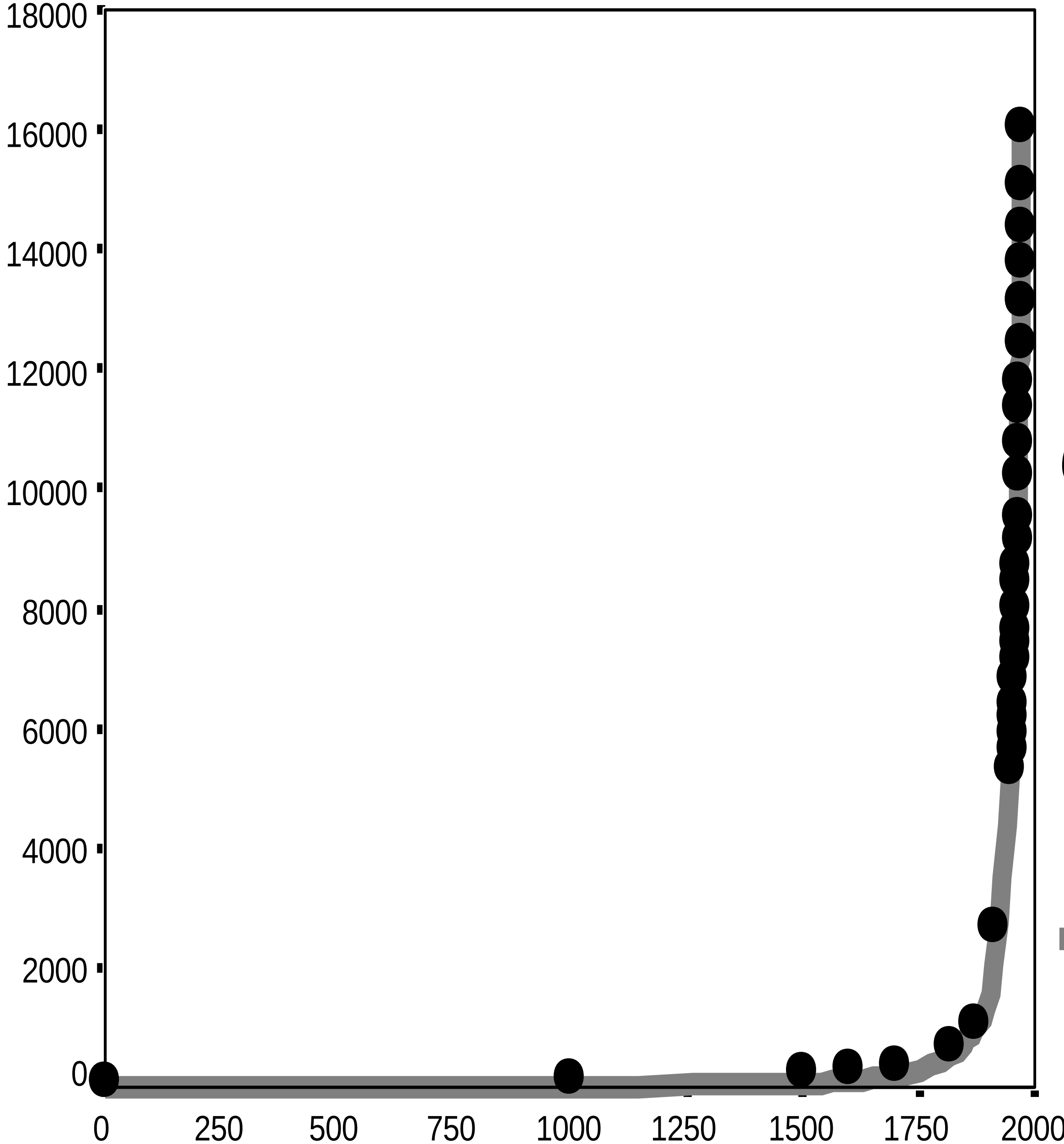
$$\frac{dX}{dt} = k_3 NX$$

$$\frac{dN}{dt} = aSN$$

$$\frac{dS}{dt} = bNS$$

where N is the world population, and

S is surplus, which is produced per person with the given level of technology over the amount, which is minimally necessary to reproduce the population with a zero growth rate.

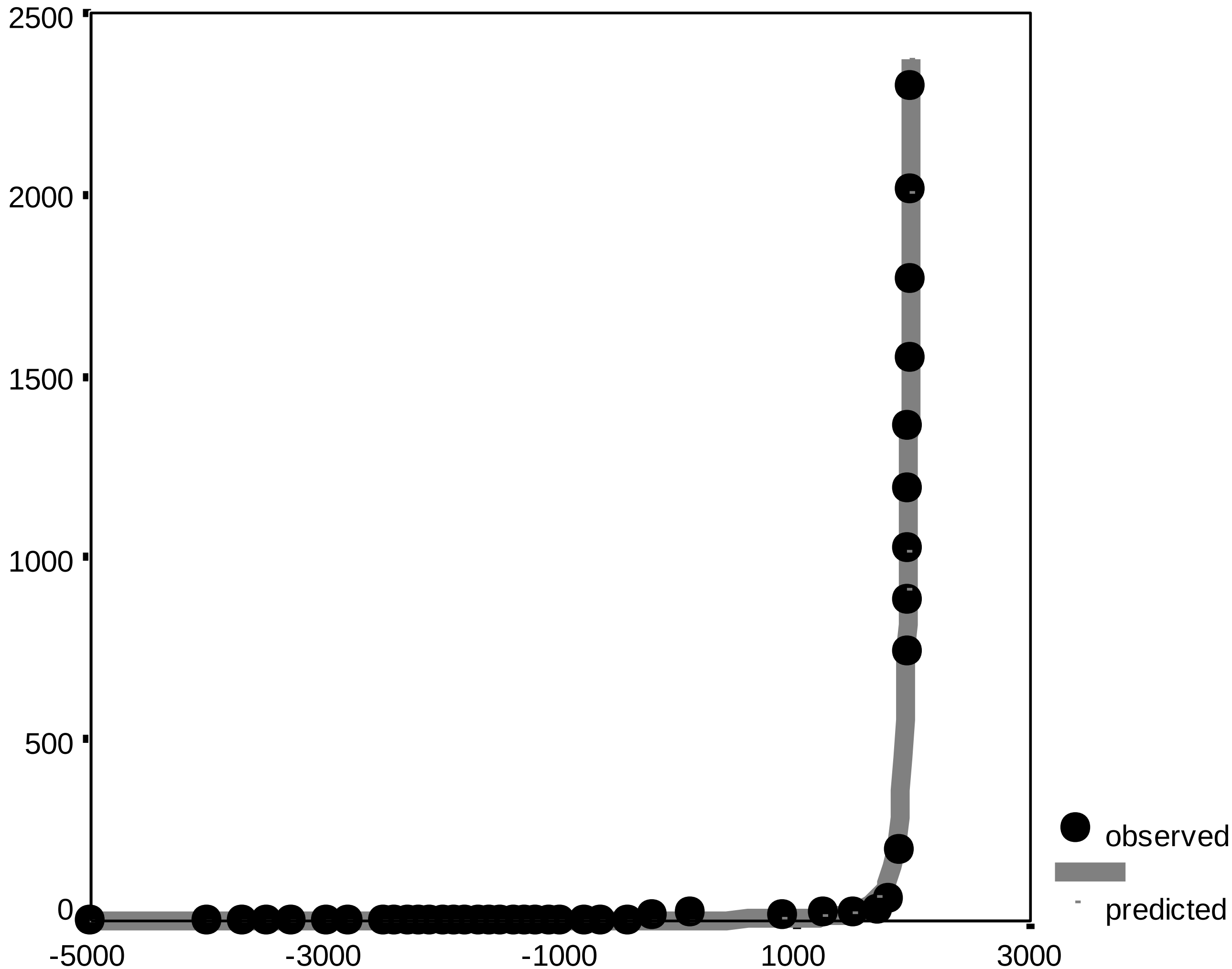


$$R^2 = .9986$$

$$G = \frac{17355487.3}{(2005.56 - t)^2}$$

2005.56 = 23 July, 2005

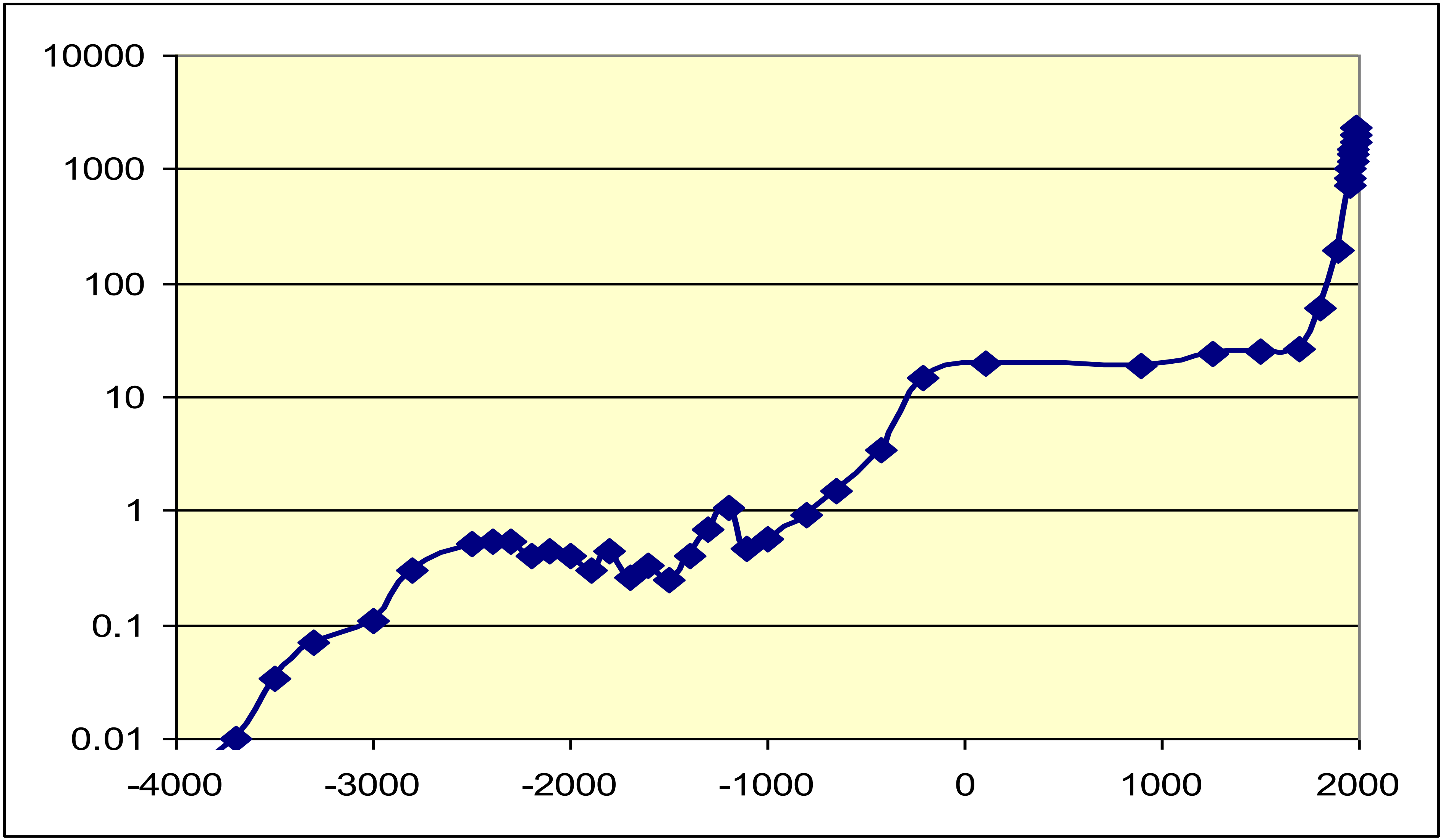
World Urban Population Dynamics (in millions), for cities with > 10000 inhabitants (5000 BCE – 1990 CE): correlation between the dynamics generated by the quadratic-hyperbolic model and empirical estimates



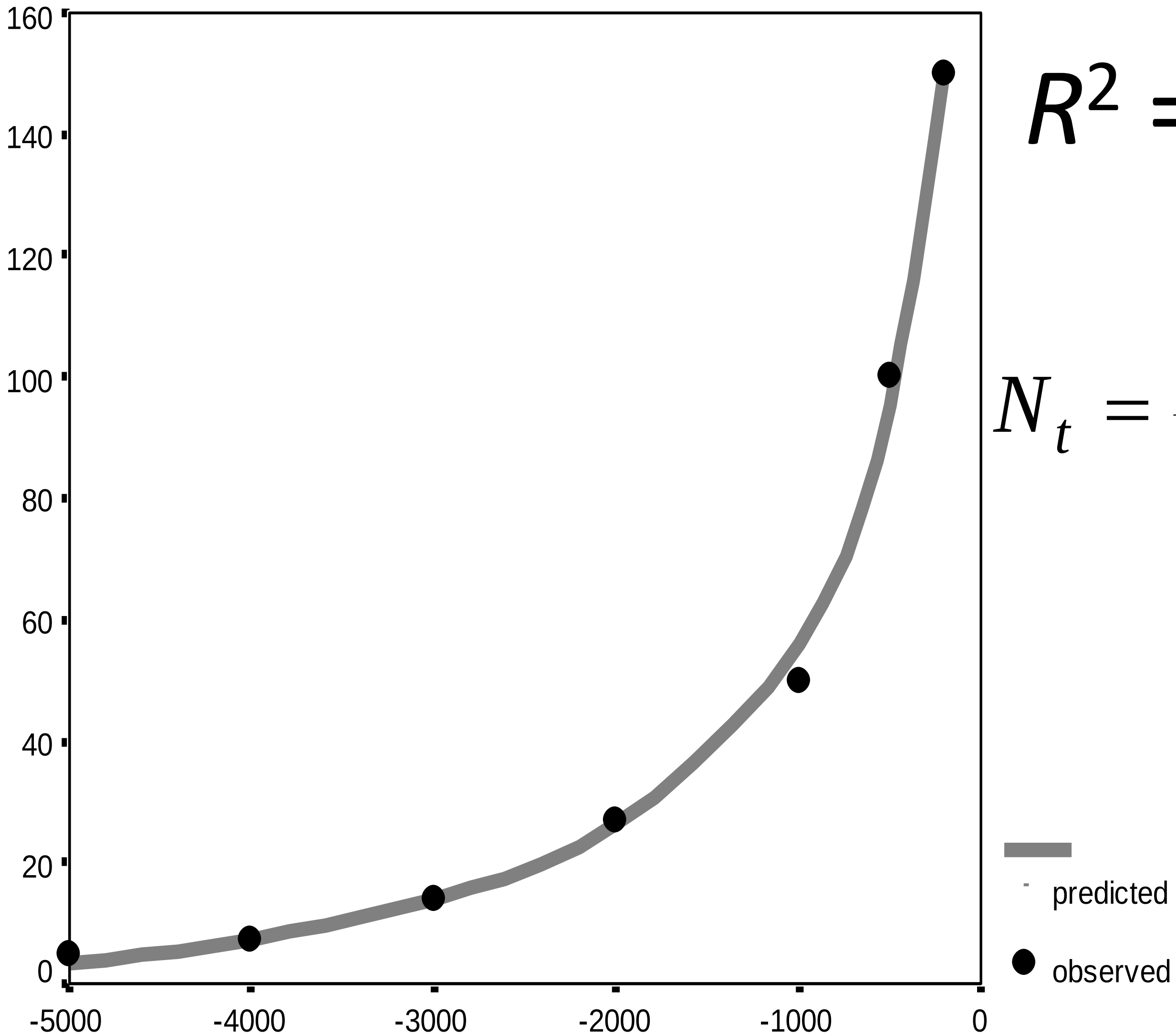
R^2 **= 0.996**

$$U_t = \frac{7705000}{(2047 - t)^2}$$

Dynamics of the World Urban Population (in millions), for cities with >10000 inhabitants (5000 BCE – 1990 CE), LOGARITHMIC SCALE



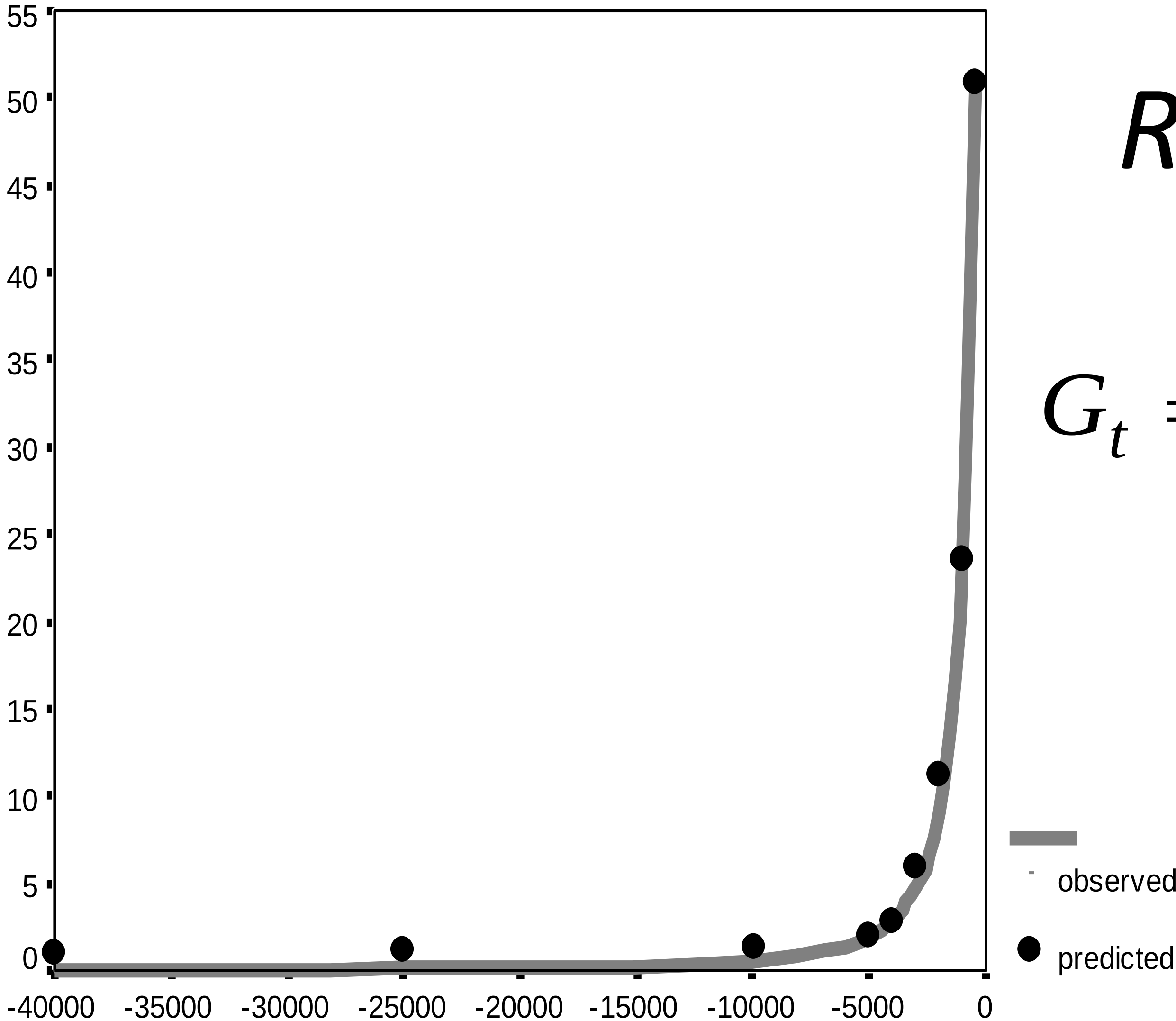
World Population Dynamics (in millions), 5000 – 500 BCE: correlation between predictions of simple hyperbolic model and empirical estimates



$$R^2 = 0.996$$

$$N_t = \frac{99674.642}{(400 - t)} - 15.29$$

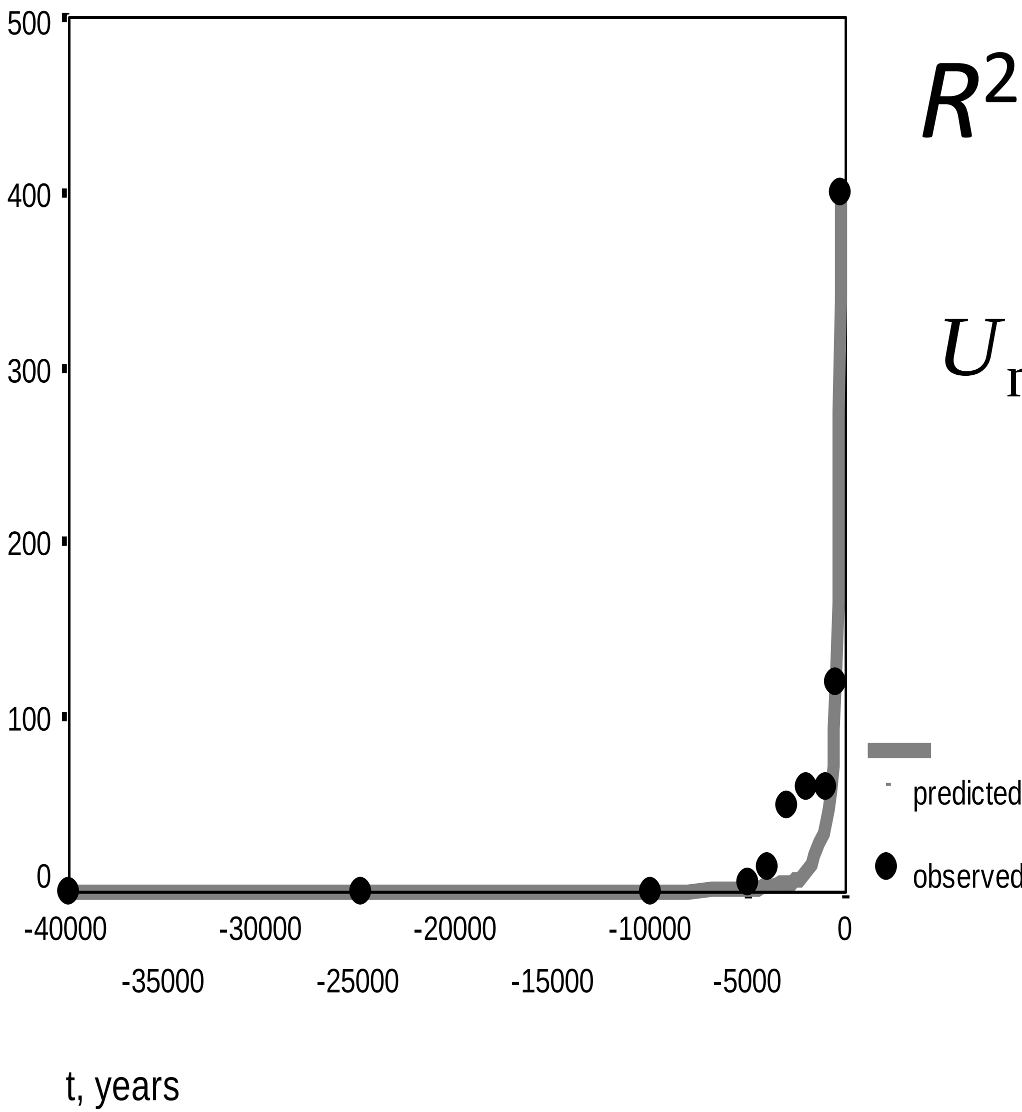
World GDP Dynamics (in billions of international 1990 dollars, PPP), 40000 – 500 BCE: correlation between predictions of quadratic-hyperbolic model and DeLong's estimates



$$R^2 = 0.998$$

$$G_t = \frac{61303619.77}{(595 - t)^2}$$

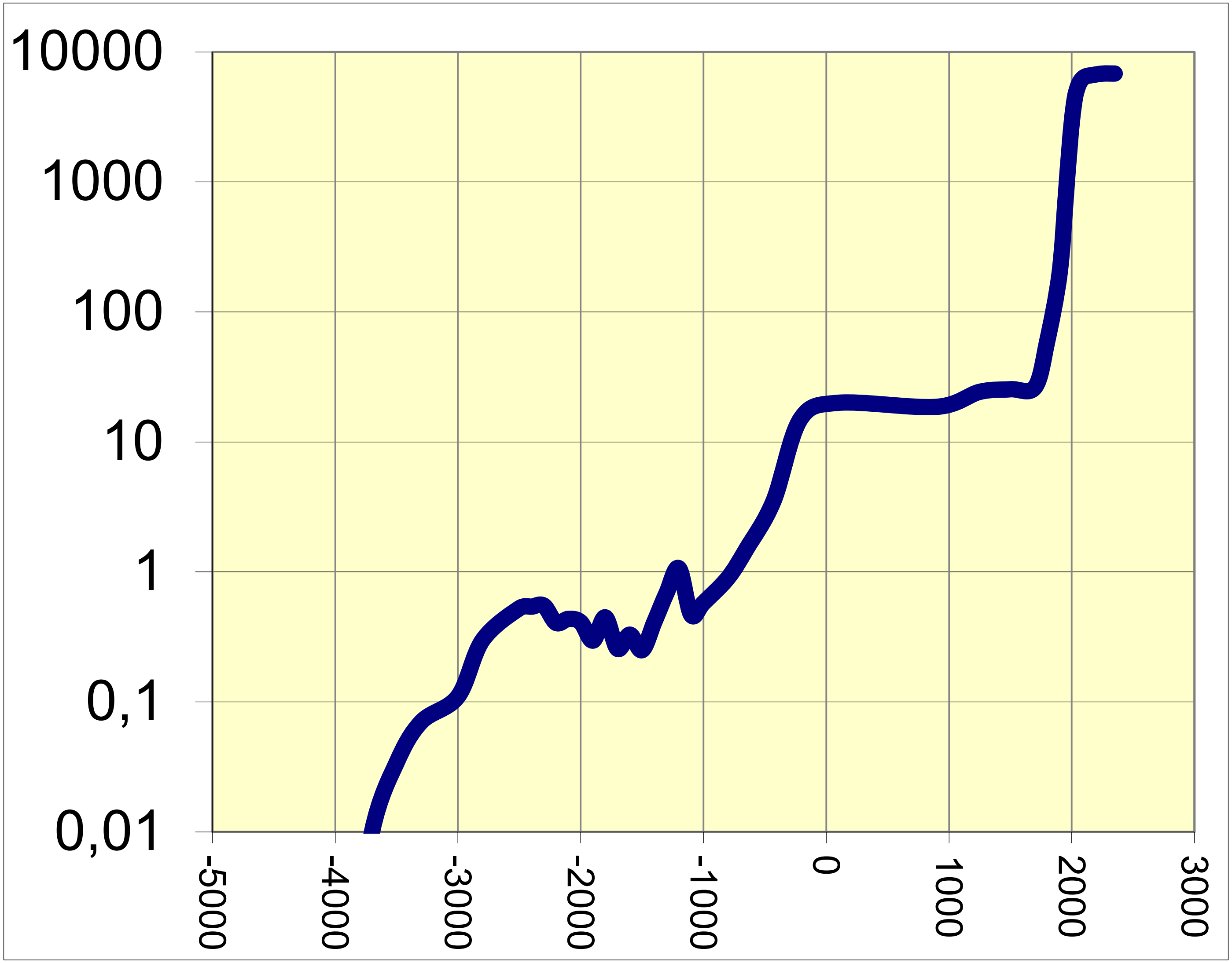
Dynamics of Population of the Largest Settlement of the World System, in thousands, 40000 – 200 BCE: correlation between predictions of quadratic-hyperbolic model and empirical estimates



$$R^2 = 0.978$$

$$U_{\max t} = \frac{56637733.865}{(175 - t)^2}$$

World Urban Population Dynamics (in millions), for cities with >10000 inhabitants (5000 BCE– 2010 CE), with a forecast till 2250, LOGARITHMIC SCALE



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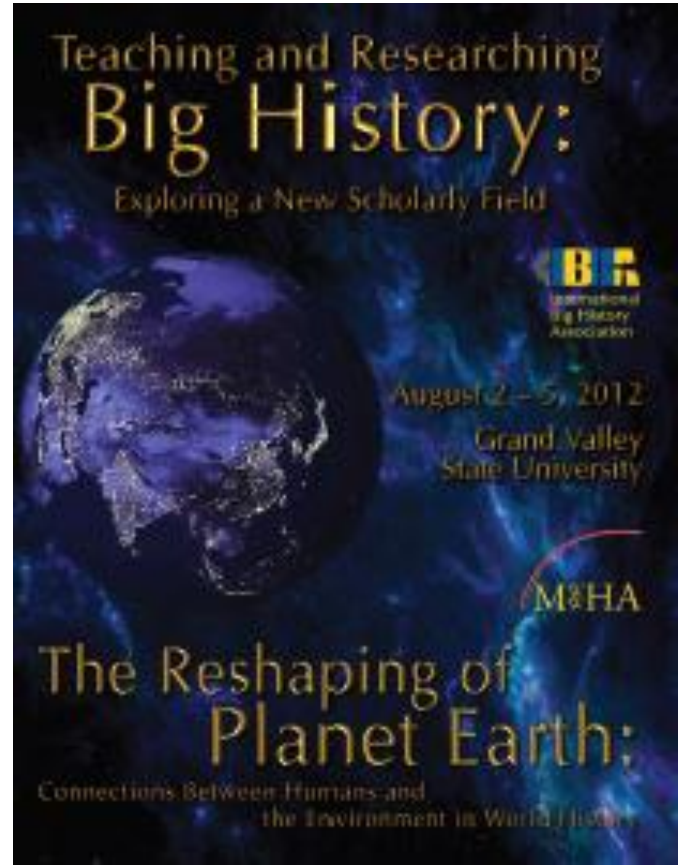

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