Fission, Fusion and Quantisation in Global Political Evolution

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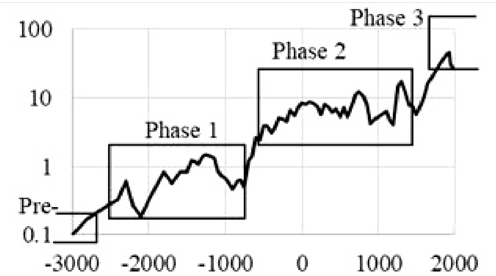
The article will show how an abstract theory of political evolution could be constructed using ideas from nonlinear dynamics. This is inspired by work on social macrodynamics that has proven the power of compact mathematical models for understanding global history. The features addressed are: quantisation of polity size; pathways of fission and fusion in response to Malthusian stress; the connections between population growth, resource pressure, conflict, and technological and institutional change.

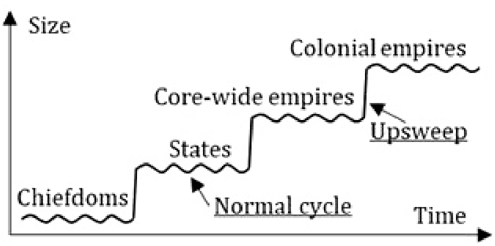
Keywords: *political evolution, macrodynamics, conflict, imperialism.*

This paper describes an approach to a mathematical model of long-term global political evolution. It is a sketch rather than a completed theory. It is inspired by the compact macromodels of global evolution developed by Korotayev, Malkov, and Khaltourina (2006a, 2006b). Such models capture the phenomenon in a few equations and variables, making them unambiguous and graspable by the human mind. The goal is deep, principled understanding, which would be supplemented by nuanced, contextual knowledge for real world situations.

Several authors have found quantisation in evolution of political institutions. Taagepera (1978) noted a step-wise growth of empire sizes. Chase-Dunn *et al.* (2010) formalised this into a 'normal cycle', during which polities fluctuate around a typical size, and 'upsweeps', when the size abruptly increases (see Fig. 1). Korotayev and Grinin (Korotayev 2006; Grinin and Korotayev 2006) state that world urbanisation consists of a series of attractors (*e.g.,* 'complex agrarian society') divided by transitional phases.



(a) % world under three largest empires



(b) Stylisation

Fig. 1. Normal cycles and upsweeps in political evolution *Sources:* Taagepera 1978; Chase-Dunn *et al.* 2010.

Henry Wright (2018) shows quantisation in the sizes of polities' largest settlements, which form discrete clusters rather than varying continuously (see Fig. 2). Fletcher (1995) also identifies quantisation of settlement sizes, which he attributes to a series of communications revolutions that made it possible to administer settlements of larger areal extent hence population, given limits on population density. Other illustrations of socio-political quantisation include Johnson on types of political official (Johnson 1982), Johnson and Earle on social evolution (Johnson and Earle 2006: 314), DeDeo on evolution of norm-observance (DeDeo 2017), Carneiro on levels of political integration (Carneiro 1987), and Korotayev and Grinin (Korotayev and Grinin 2006) on the typology of states.

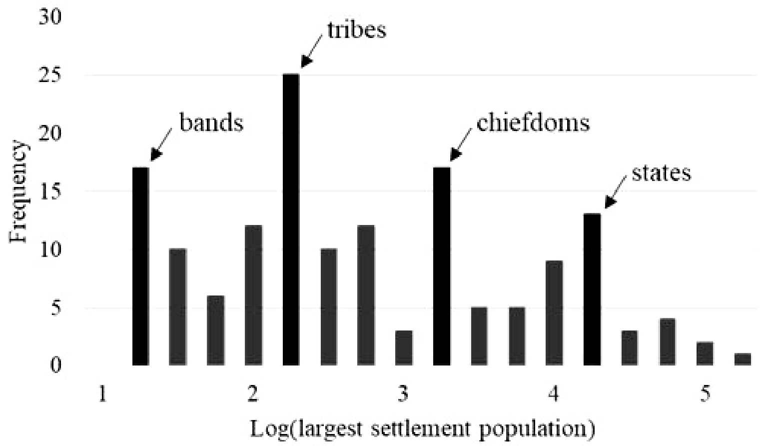


Fig. 2. Multi-modal distribution of the largest settlement sizes for a range of archaeologically studied societies

Source: Wright 2018.

Thus, the quantum nature of political evolution manifests itself in population (Wright 2018), areal extent (Taagepera 1978), settlement size / urbanisation (Fletcher 1995; Korotayev 2006), and institutional complexity (Carneiro 1987; DeDeo 2017; Johnson and Earle 2006; Korotayev and Grinin 2006).

One way into understanding this is through Graber's work on political fission and fusion among a set of societies (Graber 1995). As population increases, the societies try to maintain constant density and size by splitting and dispersing, resulting in numerical proliferation and areal expansion. If this is not possible because there is no empty territory, then, according to Graber's 'symmetry thesis', societies fuse so that any increase in density is matched by an equal and opposite decrease in the number of societies. This is a mathematisation of Carneiro's circumscription theory of state formation (Carneiro 1970), but avoids the problem highlighted by Zinkina *et al.* (Zinkina, Korotayev, and Andreev 2016) that conquest is virtually absent from village-level societies since Graber makes no claim about how societies merge, only that they do.

We can understand fusion via Turchin's idea that 'war' encourages growth of 'integrative factors', which suppress war (Turchin 2016). Plundering of land and property, suggestive of pressure on resources, are prevalent in small societies (Zinkina, Korotayev, and

Andreev 2016), and such conflict might eventually make people accept over-arching political institutions (integrative factors) that bind neighbouring societies together and settle their disputes. This shifts the focus from conquest by ambitious leaders to the mood and motivation of entire populations, that is in Tolstoy's imagery, from the Napoleons to the generalised 'swarm life' of humanity (Tolstoy 1957).

A striking feature of political quantisation is that the steps appear roughly equally spaced on logarithmic axes (see Figs 1a, 2;Korotayev 2006). It means the sizes of political units go up in a geometric progression.[1] This makes sense if *N* villages become fused into a chiefdom, *N* chiefdoms become fused into a simple state, and *N* simple states become fused into a complex state. Then the sizes of political units go up in the ratio $1:N: N^2:N^3$. Such constancy in the number of fusing units suggests that conflict, and subordination to higher political control, operate in essentially the same way at each level, so one abstract model might cover all stages of political evolution.

From these findings and theories we can synthesise the picture of Fig. 3. We imagine a region, which might be the Amazon rainforest, a continent, or the whole world. It is occupied by polities, which we will call villages. As population grows, villages become oversized, resulting in more frequent disputes (Johnson 1982) until one faction leaves and sets up a new village. This is the normal cycle, when polities fluctuate around a typical size. Eventually, the whole region is filled. Now as population grows, disputes arise between villages, which are competing for a larger share of finite resources. The resulting conflict leads to fusion of villages into chiefdoms. This is the upsweep, that is emergence of a higher-level polity much larger than any previous ones. Such fusion of villages into chiefdoms goes on across the whole region, in another normal cycle, until all villages have been absorbed. At this point, there is no more space for chiefdoms, and there is another upsweep as chiefdoms fuse into simple states, and so on. Note that the deproliferation of villages is the same as the proliferation of chiefdoms, so in general fusion of polities at one level corresponds to fission of polities of the next higher level.[2]

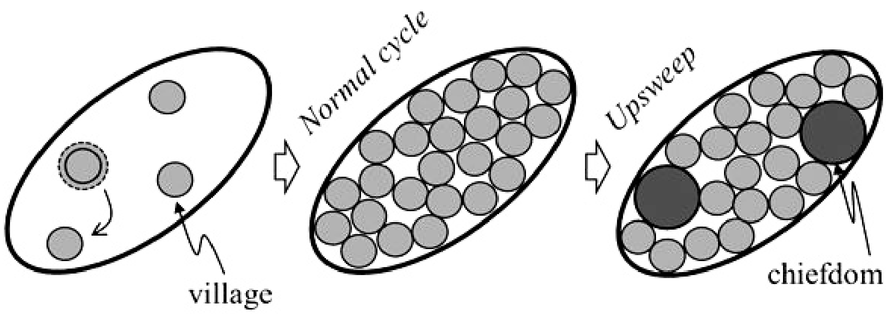


Fig. 3. Stylised view of polity fission and fusion

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[1] Figure 2 is about the largest settlement sizes not overall polity sizes. However, they are related. For an intuitive grasp consider that the 'capital' (largest settlement) provides functions for managing the entire polity, and so the size of the capital is likely to reflect the size of the polity. This is complicated by the existence, often in the New World, of administrative capitals that are distinct from the economic capital.

[2] Some fusion may be spontaneous. However, Polynesian chiefdoms are often set up by factions leaving an existing chiefdom (Kirch 2002), while many modern countries celebrate independence days, showing that they have arisen by fission as much as fusion.

We might ask how fusion resolves conflict, since the underlying problem of overpopulation and resource competition would seem to remain. The answer is that political enlargement allows economic intensification, thus supporting a higher population on the same resource base. The fused population can specialise and invest on a larger scale. An autonomous village has only simple subsistence technologies, but chiefdoms, states and modern nations have metal-working, roads and consumer electronics that are not viable for a village alone. The carrying capacity of the fused polities is more than the sum of their individual carrying capacities, and a population that exceeded carrying capacity when divided into many polities can be less than the carrying capacity when they are combined. Specialisation also applies to political officials. A chiefdom can support a small standing army, which a village cannot. Hence fusion/enlargement reduces the causes of conflict (resource pressure) as well as providing more effective methods of managing conflict.

For the purpose of abstraction, we will say that a fused population supports a higher 'technology', which may include social technologies (institutions) as well as material ones. Abstractly, a technology is an activity network or organising principle for human behaviour, whether political, economic or cultural. (A smartphone is not really metal and glass but the app developers, microwave engineers and salespeople who make it a living technology.) Networks imply a loss of autonomy or a need to fit in with others, and are resisted until population pressure makes them necessary.

To represent the above argument mathematically, we suppose that population density, *i.e.* population P divided by area A is linearly proportional to technology T (Widdowson 2020), so

$$\frac{P}{A} \sim T \qquad (1)$$

This is true of the long-run. In the short run, P/A grows logistically to the limit set by T. The linear relation between P and T is justified by the social macrodynamics work, where it leads to accurate results (Korotayev, Malkov, and Khaltourina 2006a, 2006b).

As argued above, more elaborate technologies need larger populations to support them. We can write

$$P \sim T^{\alpha} \qquad (2)$$

where α is some exponent to be determined (see Widdowson 2020) for arguments that $\alpha = 11/6$. Here we will take $\alpha = 2$, which simplifies the expressions and is close enough for qualitative discussion. We expect $\alpha = > 1$ because friction in behavioural networks means that the number of people needed to supply technology grows faster than the number of people technology can support. With $\alpha = 2$, Equation 2 becomes

$$P \sim T^2 \Leftrightarrow T \sim P^{1/2} \qquad (3)$$

In equilibrium, both Equations 1 and 3 must be satisfied. This gives

$$\frac{P}{A} \sim T \sim P^{1/2} \Rightarrow P^{1/2} \sim A \Rightarrow P \sim A^2 \qquad (4)$$

Hence, the equilibrium viable population increases with the square of the area. If we start with ν level n polities, each of area A_n and of viable population (carrying capacity) P_n, and fuse them into a level $n + 1$ polity, the area after fusion will be $A_{n+1} = \nu A_n$, while the viable population will be $P_{n+1} = \nu^2 P_n$.[3]

Let us define population pressure L (for 'load' – Graber's suggestion) as the ratio of actual population to carrying capacity. For level n *n* polities with population P, we have

$$L_n = \frac{P}{P_n} \tag{5}$$

and when ν such polities have been fused into a level $n + 1$ polity, we have

$$L_{n+1} = \frac{\nu P}{\nu^2 P_n} = \frac{1}{\nu} L_n \tag{6}$$

Thus, fusing ν polities reduces population pressure by a factor of ν. Suppose fusion occurs when population pressure reaches some threshold $\theta > 1$, while people require a level of political fusion that reduces it to a value δ. This implies the requirement

$$\frac{\theta}{\nu} \approx \delta \tag{7}$$

The Amazonian data suggest that fission occurs when population is about 50 per cent above the typical level, *i.e.* θ ‰ 1.5 (Carneiro 1970, 1987; Chagnon 1997). The figure is approximate because the threshold for fusion, a more complex process, is probably higher than that for fission. Meanwhile, Figs 1a and 2 suggest that polity sizes increase by around 10 with each level, so $\nu^2 \approx 10 \Rightarrow \nu \approx 3$. If we said the factor lay between 5 and 50, it would give ν between 2 and 7, *i.e.* fusion typically involves between 2 and 7 polities, a reasonable range. With $\theta \approx 1.5$ and $\nu \approx 3$, we find $\delta \approx 0.5$, *i.e.* fusion reduces population to about 50 per cent of carrying capacity (by increasing carrying capacity not by decreasing population). This can be expected to eliminate resource conflict and leave potential for further growth.

This result concerns long-run envelopes of social processes. The higher carrying capacity depends on specialisation, which will not emerge instantly. Conflict will first be reduced by the new political institutions (*e.g.,* chiefs or tribal councils) of the fused polity. Economic intensification will come later, and on even longer timescales there may arise group solidarity to replace coercion with co-operation in attaining social control.

We can now construct a generalised, abstract theory of political evolution. Our presentation will be neither entirely rigorous nor complete, and will leave open certain questions. It is, on the other hand, intuitive and easy to follow, and lays the ground for a more mathematically and conceptually challenging theory to come.

Quantisation emerges when a system's behavioural equations have periodic solutions. Here, we want to build a model where different levels of political complexity emerge, in a suitable phase space, as regularly spaced attracting fixed points representing periodic solutions to a set of equations (see Fig. 4), where the phase space dimensions are taken to be population density and population pressure, and remember that this describes a collection of societies not an individual society. The figure shows two possible trajectories within the phase space.

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[3] To see this, put a constant of proportionality k in the last part of Eq. 4, to give $P = kA^2$. Then we have $P_n = kA_n^2$ and $P_{n+1} = kA_{n+1}^2 = k\left(\nu A_n\right)^2 = \nu^2\left(kA_n^2\right) = \nu^2 P_n$.

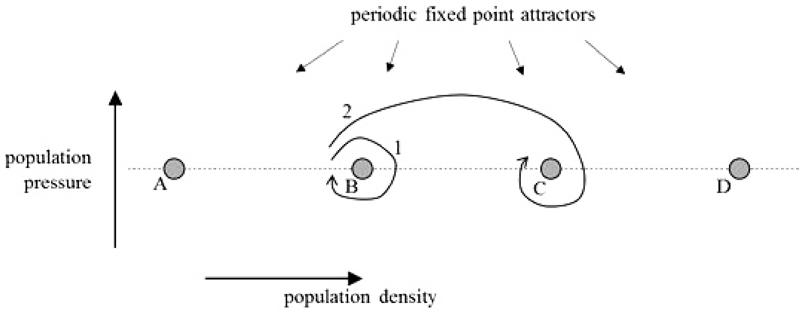

Fig. 4. Periodic attractor fixed points with possible trajectories

Trajectory 1 corresponds to the normal cycle. As population density increases, population pressure increases until societies start to split thus relieving population pressure. The resulting dispersal of societies into empty territory causes population density to stop increasing then to decrease. This further reduces population pressure so splitting ceases, which eventually reverses the decline of population density, and the whole process repeats.

Trajectory 2 represents an upsweep. In this case, population density and population pressure increase so rapidly that the collection of societies is carried away from fixed point B and becomes attracted to fixed point C. At C, the population density is higher than at B but the population pressure is the same, *i.e.* this represents the societies with the technologies necessary for higher density living.

A picture like that of Figure 4 emerges in the case of the rigid pendulum (see Figure 5).

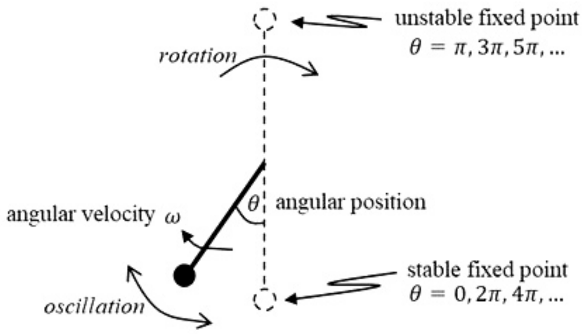

Fig. 5. The rigid pendulum

Stable positions occur when the pendulum is hanging vertically downwards, which we can see as a set of periodic fixed points at angles of 0, 2π, 4π, *etc*., *i.e.* when the pendulum has done 0, 1, 2, 3, *etc.* full revolutions. There are also unstable fixed points at angles of π, 3π, 5π, *etc*., when the pendulum is vertically upward, and these lie between the basins of attraction of the stable fixed points.

The equations that describe the pendulum are

$$\begin{aligned} \frac{d\theta}{dt} &= \omega \\ \frac{d\omega}{dt} &= -\sin\theta \end{aligned} \qquad (8)$$

where we choose units to make all proportionality constants equal to 1. Putting $\omega = 0$ and $\theta =$ any multiple of π in these equations gives $d\theta / dt = d\omega / dt = 0$, *i.e.* a fixed point. Even multiples of π are stable (when θ increases in the positive direction, $d\theta / dt$ increases in the negative direction, and vice versa), while odd multiples of π are unstable (the reverse is true).

The pendulum's phase portrait, with dimensions θ and ω, is shown in Figure 6.

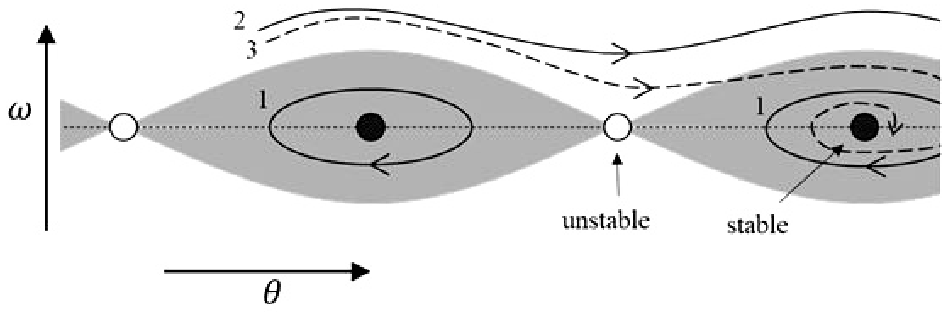

Fig. 6. Pendulum phase portrait

Trajectories 1 within the shaded regions represent the pendulum swinging back and forth. Trajectory 2 occurs when the pendulum is moving so fast that it goes over the top and rotates rather than oscillates. Thanks to friction, such a rotating pendulum would eventually slow down and oscillate around one of the fixed points, as shown by the dashed Trajectory 3.

Comparing Figures 6 and 4 shows the pendulum dynamics are what we are seeking and, given equations for political evolution structurally equivalent to Equations 8, we would have a model of political quantisation with a fission-like normal cycle and fusion-like upsweeps. One possibility is as follows

$$\frac{dP}{dt} = -C \tag{9a}$$

$$\frac{dC}{dt} = \sin P \tag{9b}$$

where C is conflict, P is population, and we define units such that all constants are unity. Although the minus sign is in a different place, these yield the same dynamics as Equations 8a and 8b.

Equation 9a shows that conflict causes population to decrease. Equation 9b says that increasing population causes conflict first to increase then decrease then increase again and so on sinusoidally.

One issue is that C can become negative, which does not seem meaningful. However, if we define $C = \ln W$ where W is, say, the rate of disputes, negative values of C mean small but still positive values of W.

A bigger issue concerns the sinusoidal oscillation, which is introduced ad hoc. We would like to obtain a model in which the sine function emerges naturally. One can rewrite Equations 9a and 9b as

$$\frac{dP}{dt} = -C \tag{10a}$$

$$\frac{dC}{dt} = X \tag{10b}$$

where X is some variable. If we have another model yielding $X = \sin P$, we would have what we want. To do this, let X be population pressure L and suppose that

$$\frac{dT}{dP} = L \tag{11a}$$

$$\frac{dL}{dP} = -T \tag{11b}$$

where T is technology. These equations simplify to

$$\frac{d^2 L}{dP^2} = -L \tag{12}$$

which has the following solution

$$L = \sin P \tag{13}$$

(neglecting constants of integration, which will introduce an arbitrary amplitude and phase but not change the dynamic).

Equation 11a shows that population pressure increases the rate at which technology grows with population. This can be understood as a Boserupian effect whereby population pressure stimulates innovation (Boserup 1965). According to Equation 11b, technology has a negative effect on the growth of population pressure with rising population. Indeed we might expect that technology should mitigate population pressure. However, we also expect technology to be positive, which means that $-T$ is always negative and population pressure will always fall with rising population. This is unrealistic. We can resolve it by introducing a logarithm to give

$$\frac{d(\ln T)}{dP} = L \tag{14a}$$

$$\frac{dL}{dP} = -\ln T \tag{14b}$$

The equations still simplify to give $L = \sin P$, but now, if T is taken relative to some baseline and can be less than 1, $\ln T$ can be negative and dL/dP can be positive.

We can get rid of the arbitrariness of introducing a logarithm by rewriting Equations 14a, 14b as

$$\frac{1}{T}\frac{dT}{dP} = L \tag{15a}$$

$$\frac{d}{dT}\frac{dL}{dP} = -\frac{1}{T} \tag{15b}$$

Equation 15a comes from applying the chain rule to Equation 14a. It shows that the *relative* rate of increase of technology with population depends on population pressure. This makes sense. The amount of innovation produced by population pressure should depend on a society's starting point, *i.e.* whether it has electricity and computers or only stone tools.

Equation 15b comes from differentiating Equation 14b with respect to T. We can rewrite it further as

$$d\left(\frac{dL}{dP}\right) = -\frac{dT}{T} \tag{16}$$

This says that the reduction in rate of growth of population pressure depends, again, on the *relative* technology change. It seems reasonable that a given technological improvement has a bigger effect if the society is starting from a lower base.

So far we have been working with total population *P*. However, our interest is in quantisation of individual polity size *S*. By Equations 9a, 9b, the stable fixed points of *P* are at $P = 0, 2\pi, 4\pi$, *etc*., an *arithmetic* progression. Yet polity sizes go up in equal multiples, a *geometric* progression. We can arrange this if *P* is related to *S* by another logarithmic relationship. Then, the equally spaced fixed points of *P* will mean that the fixed points of *S* are equally spaced on a logarithmic scale, as we find.

Suppose that *S* increases with *P* at a rate that depends on *S*. This makes sense because polities grow by fusion so the increase will be proportional to what is being added, *i.e.* to *S*. Again neglecting proportionality constants, we have

$$\frac{dS}{dP} = S \tag{17}$$

Inverting and solving, we get

$$\frac{dP}{dS} = \frac{1}{S} \Rightarrow P = \ln S \tag{18}$$

which is what we wanted. Since *P* grows with the logarithm of *S*, total population grows more slowly than individual polity size. This is right because the number of polities has decreased through fusion (Taagepera 1997), and if population is divided among fewer polities, polity size must grow faster than population.

To put all this together, our model consists of the equations

$$\frac{dS}{dP} = S \tag{19a}$$

$$\frac{1}{T}\frac{dT}{dP} = L \tag{19b}$$

$$\frac{d}{dT}\frac{dL}{dP} = -\frac{1}{T} \tag{19c}$$

$$\frac{dP}{dt} = -C \tag{19d}$$

$$\frac{dC}{dt} = L \tag{19e}$$

These produce the result that the stable fixed points of political evolution occur at $\ln S = k\pi$ where $k \in \mathbb{Z}$, *i.e.* political evolution is quantised around a set of attractor polity types whose sizes increase geometrically.

Further development is needed. To reproduce history, whereby societies oscillate around one polity type before transitioning to another polity type, we must extend the model with (1) a driving force – to push the collection of societies out of its existing basin of attraction – and (2) a friction force – to cause it to settle around the new attractor. These forces should vary over time. The equations also need units and constants of proportionality, and they could be modified in other ways, with different variables and relationships. Adding a third dimension to *P* and *C* would make possible chaotic dynamics, which might be more realistic than periodic oscillations.

A compact mathematical model of global political evolution is a desirable goal that would shed light on contemporary issues. These include: the future of the European Union, an attempt at political fusion arising from twentieth century conflict; China's Belt and Road Initiative, linking resource pressure, technology growth and political expansion; and generally the prospects for peace and unity in a multipolar world.

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